Systematic behaviours of different quantities related to sequential prompt emission in fission

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Abstract. The deterministic modelling of sequential prompt emission in fission based on recursive equations of residual temperature was applied to numerous fission cases. This fact emphasized interesting systematic behaviours of quantities characterizing the residual fragments and the sequential emission. General forms of residual temperature distributions for each emission sequence are determined on the basis of these systematics, having as application the inclusion of sequential emission into the Los Alamos model. The systematic behaviours can be also used to obtain indicative values of different average quantities in the absence of any prompt emission model.

1 Brief mentions about the modelling

A deterministic modelling of sequential prompt emission in fission was recently developed (see Ref.[1] for details). This treatment is based on recursive transcendent equations of the nuclear temperature of residual fragments. By solving such equations for each emission sequence "k" corresponding to each initial (pre-neutron) fragment A, Z at each TKE value of the fragmentation and TKE ranges, multi-parametric matrices of different quantities –generically labelled $q_k(A, Z, TKE)$- characterizing the initial and residual fragments and the prompt emission are obtained (e.g. nuclear temperature $T_k(A, Z, TKE)$ and excitation energy $E_k(A, Z, TKE)$, average prompt neutron energy in the centre-of-mass frame $<\epsilon>_k(A, Z, TKE)$ etc.). These quantities appear with the probability expressed by the fragment distribution $Y(A, Z, TKE)$.

The sequential emission modelling was applied to a large number of fission cases (benefiting of reliable experimental $Y(A, TKE)$ distributions) which includes the spontaneous fission of ^{252}Cf and ^{236,238,240,242,244}Pu, the thermal neutron induced fission of ^{233,235}U and ^{239}Pu and the fast neutron induced fission of ^{234,238}U and ^{237}Np at incident energies below the threshold of the second chance fission. I.e. a total number of 49 fission cases.

For each fissioning nucleus the initial fragmentation range was deterministically constructed as in the Point-by-Point (PbP) treatment (see Ref.[2] and references therein). I.e. a large fragment mass range was considered (with A going from symmetric fission up to a very asymmetric split). Five charge numbers Z were taken for each A as the nearest integer values above and below the most probable charge $Zp(A) = Z_{UCD}(A) + \Delta Z(A)$. A large TKE range was taken for each fragmentation, e.g. TKE from 100 to 200 MeV, with a step size of 5 MeV.

The same method of total excitation energy (TXE) partition based on modelling at scission which is used in the PbP treatment (Ref.[2] and references therein) is employed in this model, too.

Different prescriptions concerning the compound nucleus cross-section of the inverse process of neutron evaporation $\sigma_c(\epsilon)$ and the level density parameter of initial and residual fragments were used.

2 Systematic behaviours

The large number of studied cases has emphasized behaviours and correlations between different average quantities, briefly mentioned in the following.

It was ascertained that for each emission sequence the ratios of the average residual temperature to the initial one $<T_k>/<T_i>$ is almost the same for all studied cases irrespective of the prescriptions used for $\sigma_c(\epsilon)$ and the level density parameters. The upper part of Fig. 1 shows the ratios $<T_k>/<T_i>$ as a function of $<TXE>$ plotted with different coloured symbols corresponding to the emission sequences with k from 1 to 5 and to all sequences (black stars). The constant values approximating the ratios resulting from model calculations are represented by horizontal lines plotted with the same colour as the respective symbol. The ratios of the average residual energy to the initial excitation energy $<E_k>/<E^*>$ exhibit a similar behaviour as it can be seen in the lower part of Fig. 1. In both cases the constant values of the ratios for k =1 and 2 corresponding to the light and heavy fragment groups are equal. The values of the residual temperature and energy ratios of all emission sequences are just the mean of the constant values for the sequences k = 1 and 2, because the first two emission sequences take place for the majority of initial fragments at a great part of TKE values. For k > 2 the constant values of the ratios $<T_k>/<T_i>$ and $<E_k>/<E^*>$

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for the light fragments are higher than those of the heavy fragments, due to the larger difference between the residual energies of light and heavy fragments ΔE_k = $<E_k>_L$ - $<E_k>_H$ (which is of about 1.5 – 2 MeV) compared to k = 1 and 2 (less than 1 MeV).

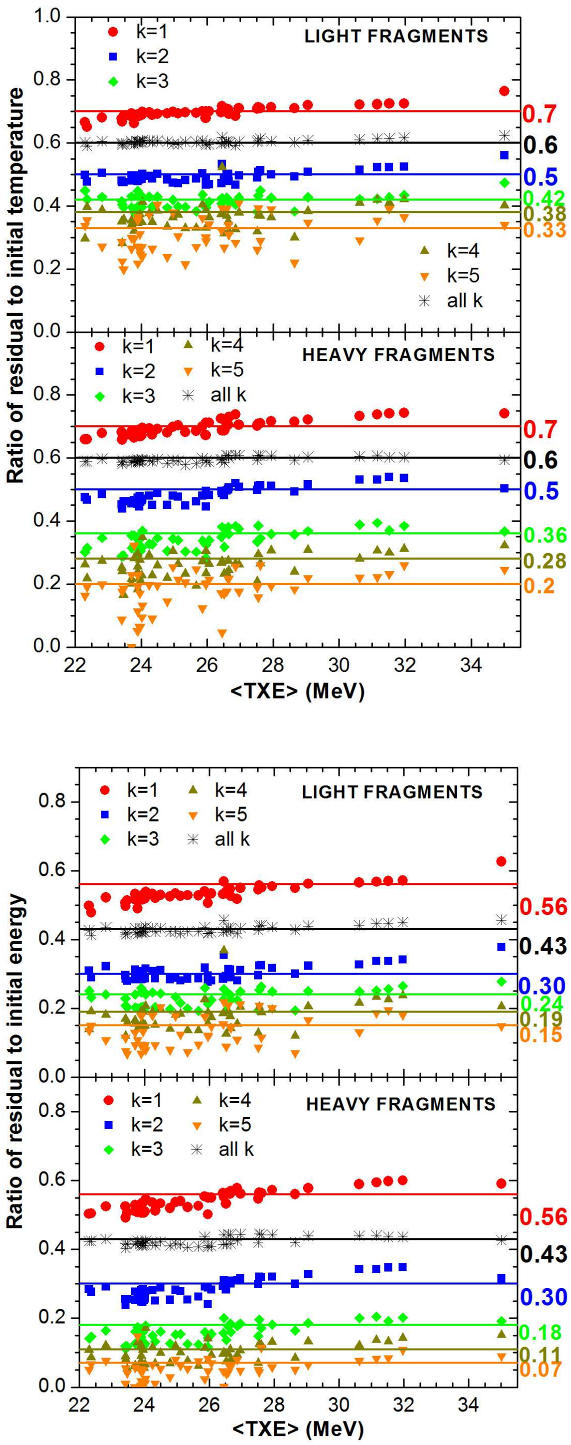

Figure 1. The ratios of residual temperature (upper part) and residual energy (lower part) to thoses of initial fragments corresponding to each sequence k (different coloured symbols) and to all emission sequences (black stars) and the constant values approximating these ratios (horizontal lines plotted with the same colour as the respective symbol and the values indicated in the right side of each frame).

A linear correlation between the average centre-of-mass energy of each emitted neutron $<\epsilon>_k$ and the corresponding average residual temperature $<T_k>$ is observed, see Fig. 2, as well as between $<\epsilon>_k$ and the root-square of $<E_k>$. The slopes of these linear dependences allow the determination of global values for the total average level density parameter of the light and heavy fragment groups. E.g. the global values $<a>_L$ = 12 MeV^{-1} and $<a>_H$ = 11.37 MeV^{-1} obtained under the level density parameter prescription of back shift Fermi-gas model are in good agreement with the values resulting from calculations.

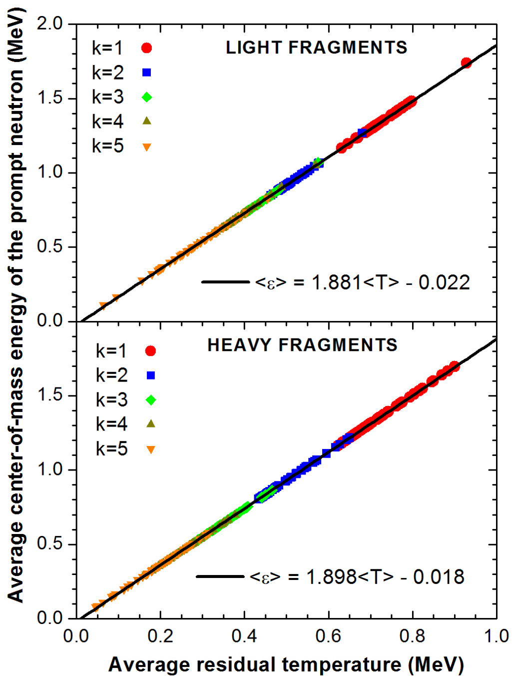

Figure 2. Average centre-of-mass energy of each emitted neutron $<\epsilon>_k$ (different symbols for k from 1 to 5) as a function of the average residual temperature $<T_k>$. The linear fits are given with black lines.

Linear dependences of the average energy carried away per each emitted neutron ($<\eta_k>$ = $<\epsilon>_k$+$<Sn>_{k-1}$) on the average neutron separation energy from the precursor fragment and of the average energy carried away per neutron corresponding to all emission sequences $<\eta>$ on the total average residual temperature $<T>$ and level density parameter $<a>$ are obtained, too.

The systematic behaviours mentioned above can be used to obtain indicative values of different prompt emission quantities in the absence of any prompt emission model. For instance, if the average temperatures of initial fragments are known then the average prompt neutron energy in the centre-of-mass frame can be obtained from the linear fit of Fig. 2 using the residual temperature values (horizontal lines) from the systematic behaviours given in Fig. 1. Two examples for ^{252}Cf(SF) with the average excitation energies of initial fragments $<TXE>$ = 35.01 MeV, $<E^*>_L$ = 19.97 MeV and $<E^*>_H$ = 15.04 MeV are given: a) using the equivalent initial temperature in which the average level density parameter is approximated by <a>=

$A_0/11$ MeV^{-1}, the value $<\epsilon>$ = 1.38 MeV is obtained, in agreement with the experimental data; b) considering the average initial temperatures $<T_i>_{L,H}$ based on the average level density parameters given by the super-fluid model $<a_L>$ = 13.55 MeV^{-1} and $<a_H>$ = 12.76 MeV^{-1}, the values $<\epsilon>_L$ = 1.43 MeV, $<\epsilon>_H$ = 1.27 MeV and $<\epsilon>$ = 1.36 MeV are obtained, also in reasonable agreement with the experimental data and model calculations.

3 Inclusion of sequential emission into the Los Alamos model

The most important feature of the most probable fragmentation approach, i.e. the Los Alamos (LA) model, consists of the global treatment of sequential emission using a residual temperature distribution $P(T)$ on which the centre-of-mass energy spectrum at a given residual temperature is integrated. Up to now the maximum temperature T_{max} of the triangular form of $P(T)$ was taken equal to the nuclear temperature of initial fragments [2] - [4]. I.e. either an equivalent initial temperature of both fragments

$$T_{max} =< T_i >_{equiv}= \sqrt{< TXE > / < a >} \qquad (1)$$

as in the LA model of Madland and Nix [3] (including the subsequent improvements) or different initial temperatures of complementary fragments as in the LA model of Madland and Kahler [4]:

$$T_{max} =< T_i >_{L,H}= \sqrt{< E^*_{L,H} > / < a_{L,H} >} \qquad (2)$$

In the PbP model ([2] and references therein) T_{max} is taken as the temperature of each initial fragment of the fragmentation range at each TKE value:

$$T_i(A, Z, TKE) = \sqrt{E^*(A, Z, TKE)/a(A, Z, TKE)} \qquad (3)$$

Taking into account the systematic of the residual temperature ratio corresponding to all emission sequences $<T>/<T_i>$ = 0.6 (black stars in Fig. 1), a triangular $P(T)$ with T_{max} related to the initial temperature (T_{max} = 0.9 $<T_i>$) was proposed in Ref.[1] and it was successfully used in the PbP model, too (see Ref.[1] for details).

The constant values of the residual temperature ratios corresponding to each emission sequence given in Fig. 1, i.e. $<T_k>/<T_i>=r_k$, allow to define a residual temperature distribution for each emission sequence $P_k(T)$ with the maximum temperatures related only to the temperature of initial fragments by the relation:

$$T^{(k)}_{maxL,H} = (3/2)r_k < T_i >_{L,H} \qquad (4)$$

in which r_k are the constant values indicated by horizontal lines in Fig. 1.

Consequently the LA model can be extended with the inclusion of sequential emission. The centre-of-mass energy spectrum of each neutron successively emitted from the light or heavy fragment is given by the following equation in which the index L, H is omitted:

$$\Phi_k(\epsilon) = \epsilon\sigma_c(\epsilon) \int_0^{T^{(k)}_{max}} K_k(T)P_k(T)exp(-\epsilon/T)dT \qquad (5)$$

where the upper limit of the integral $T^{(k)}_{max}$ is given by Eq. (4) and the normalization constant is

$$K_k(T) = \left[\int_0^\infty \epsilon\sigma_c(\epsilon)exp(-\epsilon/T)d\epsilon \right]^{-1} \qquad (6)$$

The prompt neutron spectrum in the laboratory frame (PFNS) of the k-th prompt neutron (isotropically emitted in the centre-of-mass frame of the mother fission fragment) is given by the following equation in which the index L, H is omitted, too:

$$N_k(E) = \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \frac{\Phi_k(\epsilon)d\epsilon}{4\sqrt{E_f\epsilon}} \qquad (7)$$

where $\Phi_k(\epsilon)$ is given by Eq.(5) and E_f is the kinetic energy per nucleon (the same for all sequences).

The LA model with sequential emission requires as input parameters only the average temperatures of initial fragments (based on the average excitation energies and level density parameters) and the average $<TKE>$. This model can be also used with different prescriptions concerning $\sigma_c(\epsilon)$ (e.g. provided by optical model calculations, analytical expressions etc.), the TXE partition giving $<E^*>_{L,H}$ (different methods), the level density parameters of initial and residual fragments (e.g. non-energy dependent from systematics, or energy-dependent provided by the super-fluid model etc.).

Prompt neutron spectrum results of the LA model with sequential emission are illustrated for ^{235}U(n_{th},f) taken as an example. In this calculation the following prescriptions were used: $\sigma_c(\epsilon)$ from optical model calculations with the parameterisation of Becchetti-Greenlees, the TXE partition based on modelling at scission usually employed in the PbP model [2] and level density parameters provided by the super-fluid model.

Fig. 3 shows the centre-of-mass energy spectrum $\Phi_k(\epsilon)$ of the first three neutrons (plotted with different dashed and dotted lines) successively emitted from the light and heavy fragment (upper and lower part, respectively) calculated according to Eq.(5), as well as the total centre-of-mass energy spectrum (solid red line) which describes very well the experimental data recently measured by Göök et al. [5]. The total PFNS in the laboratory frame (thick solid red line) together with the contributions of the first (dashed lines), second (dash-dotted lines) and third neutron (dotted lines) emitted from the light fragment (blue colour) and heavy fragment (green colour) and the contribution of all emitted neutrons (thin solid lines) are plotted in Fig. 4. For a better visualization, the high and low energy parts of the spectrum are focused in separate frames. As it can be seen the total PFNS is obtained in good agreement with the experimental data (different symbols) measured in the last decade.

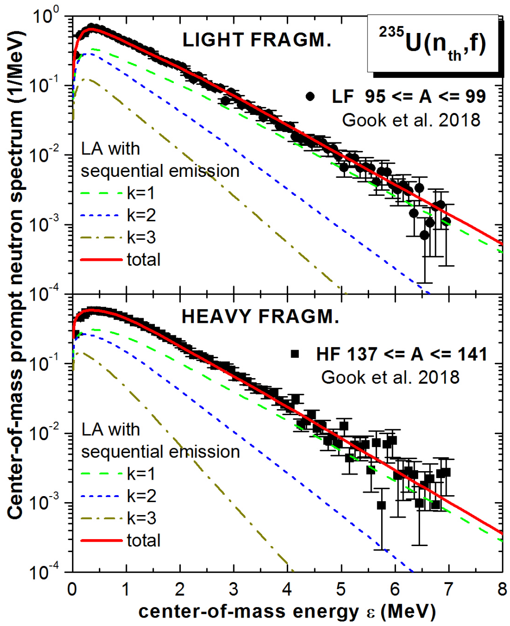

Figure 3. Prompt neutron spectra in the centre-of-mass frame corresponding to the light and heavy fragment group (upper and lower part, respectively): The result of the LA model with sequential emission: $\Phi_k(\epsilon)$ of the first three emitted neutrons (different dashed and dotted lines) and the total $\Phi_k(\epsilon)$ (red solid line) describing very well the experimental data of Göök et al.[5] (symbols).

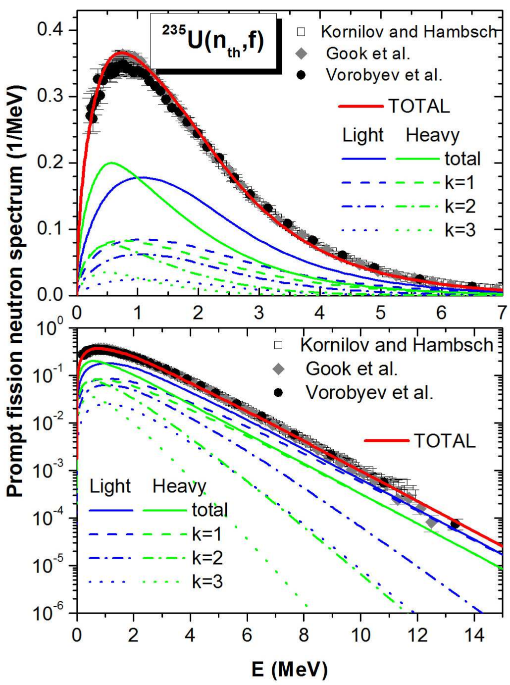

Figure 4. PFNS result of the LA model with sequential emission: spectra of each emitted neutron (different dashed or dotted lines) and of all emitted neutrons (thin solid lines) from the light (blue colour) and heavy (green colour) fragments and the total spectrum (thick solid red line) describing well the experimental data (different symbols).

4 Conclusions

The deterministic modelling of sequential emission applied to 49 fission cases allowed to obtain systematic behaviours and correlations between different average quantities characterizing the initial and residual fragments and the prompt emission.

1. The residual temperature and energy ratios corresponding to each emission sequence $<T_k>/<T_i>$, $<E_k>/<E^*>$ and to all emission sequences ($<T>/<T_i>$, $<E>/<E^*>$) are almost the same (constant values) for all investigated fission cases, irrespective of the prescriptions used for $\sigma_c(\epsilon)$ and the level density parameters of the initial and residual fragments.

2. The constant values of the temperature ratios $<T_k>/<T_i> = r_k$ (e.g. $r_1 = 0.7$ and $r_2 = 0.5$ for both light and heavy fragment groups, r_3=0.42 for light fragments and $r_3 = 0.36$ for heavy fragments, etc.) allow to define a residual temperature distribution for each emission sequence $P_k(T)$ with the maximum temperature related only to the initial temperature $T^{(k)}_{max} = (3/2)r_k<T_i>$, having as application the inclusion of sequential emission into the Los Alamos model.

3. The constant ratios $<T>/<T_i>$ and the linear behaviour of the average centre-of-mass energy of prompt neutrons $<\epsilon>_{L,H}$ as a function of residual temperature $<T>$ allow to obtain indicative values of different prompt emission quantities in the absence of any prompt emission model.

4. The linear dependences of $<\epsilon>_k$ on $<T_k>$ and on the root-square of $<E_k>$ are the same for all emission sequences. Almost linear dependences of the energy carried away by each emitted neutron $<\eta_k>$ on the neutron separation energy from the precursor $<Sn>_{k-1}$, on the residual temperature $<T_k>$ and on the average level density parameter of each residual fragment group are established, too.

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