Systematic behaviours of different quantities related to sequential prompt emission in fission

Anabella TUDORA

University of Bucharest, Faculty of Physics

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Content of this presentation

- **Deterministic treatment of sequential prompt emission** in fission applied to 49 fission cases (SF, (n_{th},f), (n,f) at En < second fission chance)

- **Systematic behaviours** of

- ratios of residual temperature corresponding to each sequence
- ratios of residual energies corresponding of each sequence
- center-of-mass energy of each emitted neutron
- energy carried away per each emitted neutron

- **Applications of systematic behaviours**

- inclusion of sequential emission into the Los Alamos model
- indicative values of prompt emission quantities in the absence of any prompt emission model

- **Conclusions**

A **deterministic modeling of sequential emission in fission**, described in *Ref. A.Tudora et al. Eur.Phys.J A, 54 (2018) 87* is applied to 49 fission cases covering a large range of nuclei and TXE values. They are:

- SF: ^{252}Cf(SF), ^{236,238,240,242,244}Pu(SF)
- (n_{th},f): ^{235}U(n_{th},f), ^{239}Pu(n_{th},f) and ^{233}U(n_{th},f)
- (n,f) below the threshold of the second chance fission:
  - ^{237}Np(n,f) at 12 En going from 0.3 and 5.5 MeV
  - ^{238}U(n,f) at 14 En going up to 5.5 MeV
  - ^{234}U(n,f) at 14 En ranging from 0.2 to 5 MeV

These nuclei benefit of reliable experimental Y(A,TKE) data, the majority of these experim. distributions were measured at JRC-Geel, Belgium except Pu(SF) – data Dematte et al., ^{233}U(n_{th},f) – data Surin (EXFOR)

The investigation of many fission cases allowed to determine interesting systematic behaviours and correlations

Fragmentation range of initial fragments (before prompt emission)

deterministically constructed as following:

- **A - from symmetric fission up to a very asymmetric split**
- **5 Z for each A - the nearest integers above and below**

$$Zp(A)=Z_{UCD}(A) + \Delta Z(A)$$

$^{233,235}U(n_{th},f)$, $^{239}Pu(n_{th},f)$, $^{252}Cf(SF)$, $^{236-244}Pu(SF)$ → $\Delta Z(A)$, rms(A) (Wahl)

$^{237}Np(n,f)$, $^{234,238}U(n,f)$ → mean values $\Delta Z(A)=|0.5|$, rms=0.6 for all A

- **TKE range (e.g. 100 – 200 MeV) with a step size of 5 MeV is taken for each {A, Z}**

TXE partition → modeling at scission → extra-deformation energy ΔE_{def} and the partition of available excit. energy at scission considering statistical equilibrium at scission and level density of fragments in the FG regime

- **Compound nucleus cross-section of the inverse process of neutron evaporation from initial and residual fragments $\sigma_c(\varepsilon)$:**
  an analytical expression depending on A and the s-wave neutron strength function

- **Level density parameter of initial and residual fragments**
  provided by the Egidy-Bucurescu systematic for BSFG

For each <u>initial fragment</u> A, Z, TKE , by solving the successive equations

$$\boxed{\overline{E_r}^{(k-1)} - S_n^{(k-1)} - <\varepsilon>_k = a_k T_k^2}$$

different quantities q_k(A,Z,TKE) for each sequence "k" are obtained,
e.g. T_k(A,Z,TKE), Er_k(A,Z,TKE), $<\varepsilon>_k$(A,Z,TKE)
η_k(A,Z,TKE) = $<\varepsilon>_k$(A,Z,TKE) + Sn_{k-1}(A,Z,TKE) (energ.carried away/n)

They appear with a probability expressed by the Y(A,Z,TKE) distribution

<u>Average values corresponding to each emission sequence</u>:

$$\boxed{\langle q_k \rangle = \sum_{A,Z,TKE} q_k(A,Z,TKE)Y(A,Z,TKE) \Big/ \sum_{A,Z,TKE} Y(A,Z,TKE)}$$

by summing separately for the light and heavy groups or over all fragments

<u>Total average quantity</u>

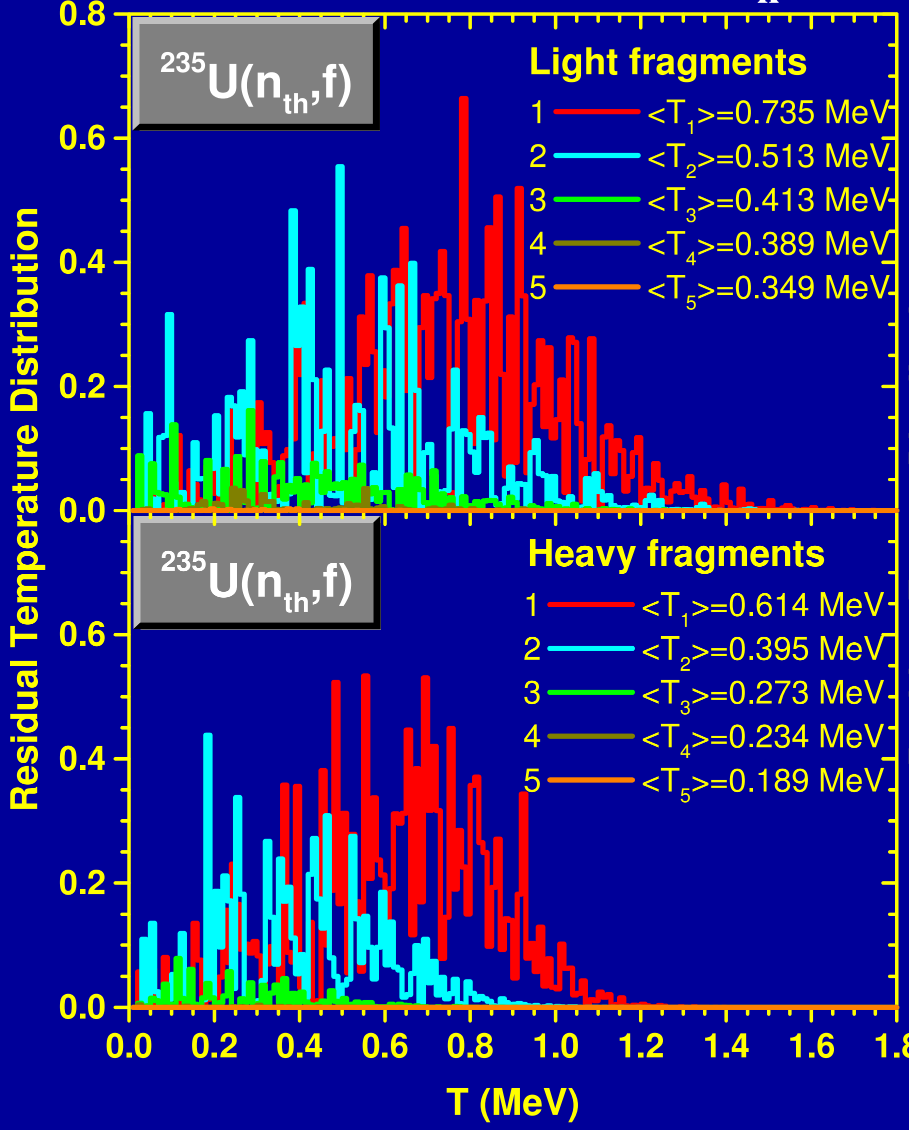
(corresponding to the sum of the distributions following the emission of each neutron)

$$\boxed{\langle q \rangle = \sum_{k=1}^{n} \langle q_k \rangle Pn_k \Big/ \sum_{k=1}^{n} Pn_k}$$

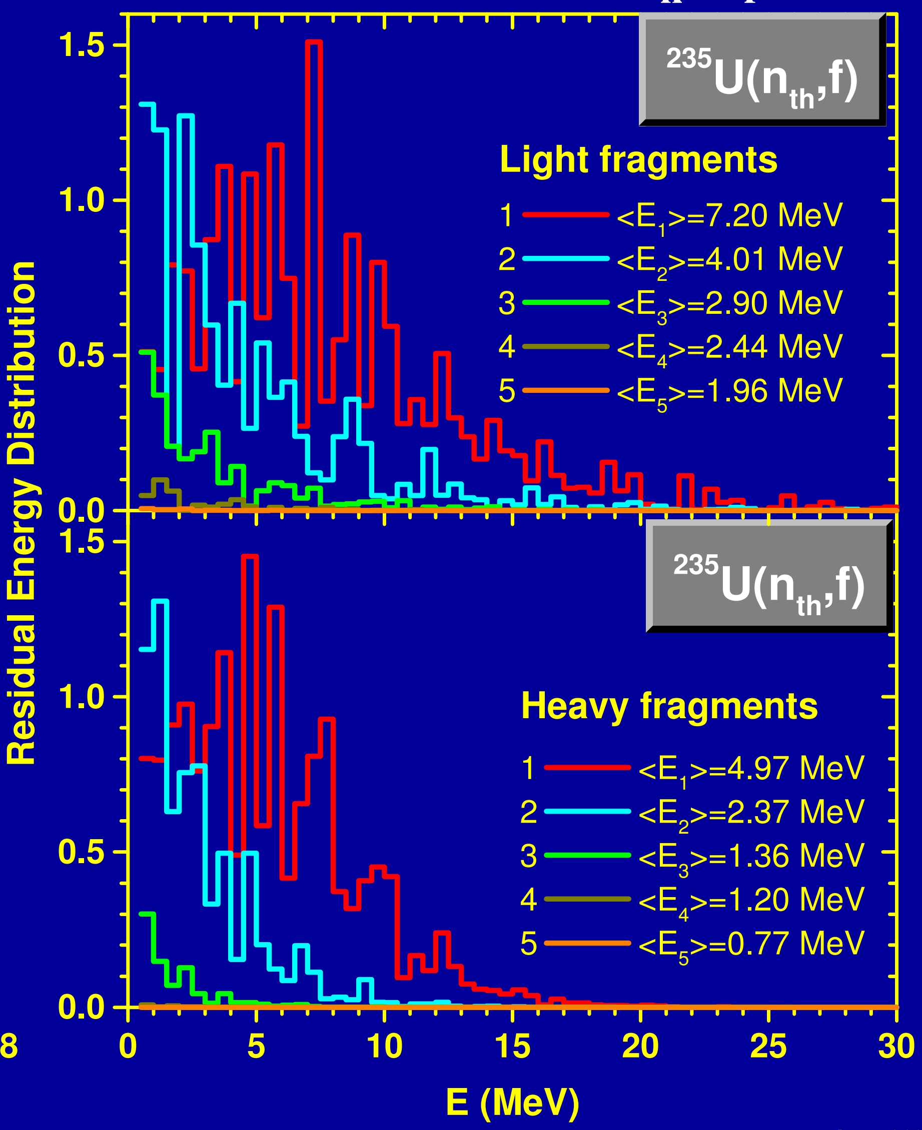
Pn_k: the probability for emission of the 1-st, 2-nd 3-rt, 4-th, …, k-th neutron.
Note, <u>to be not confounded with</u> P(ν) the probability for emission of one, two, three… neutrons

Example of distributions for each emission sequence

Residual temperature $P_k(T)$



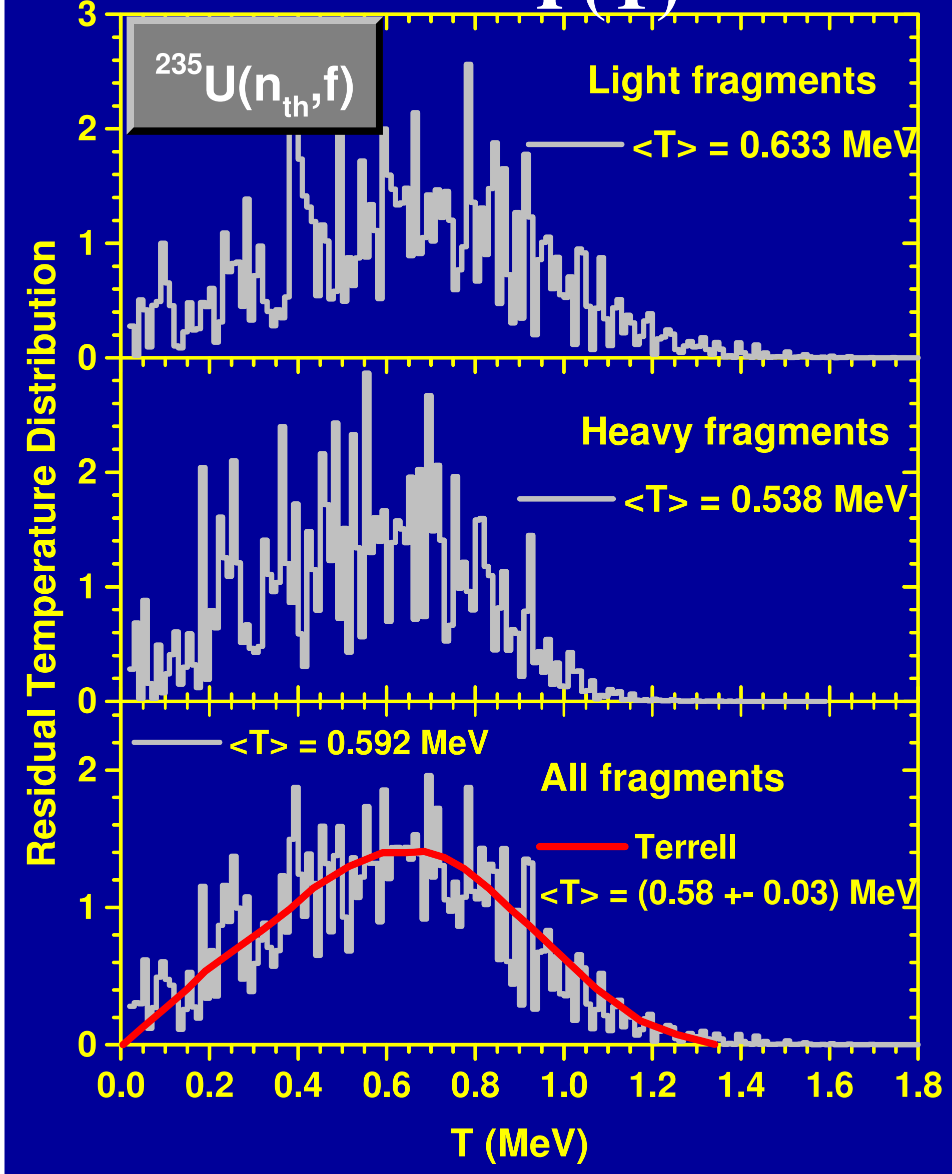
Residual energy $P_k(E_r)$



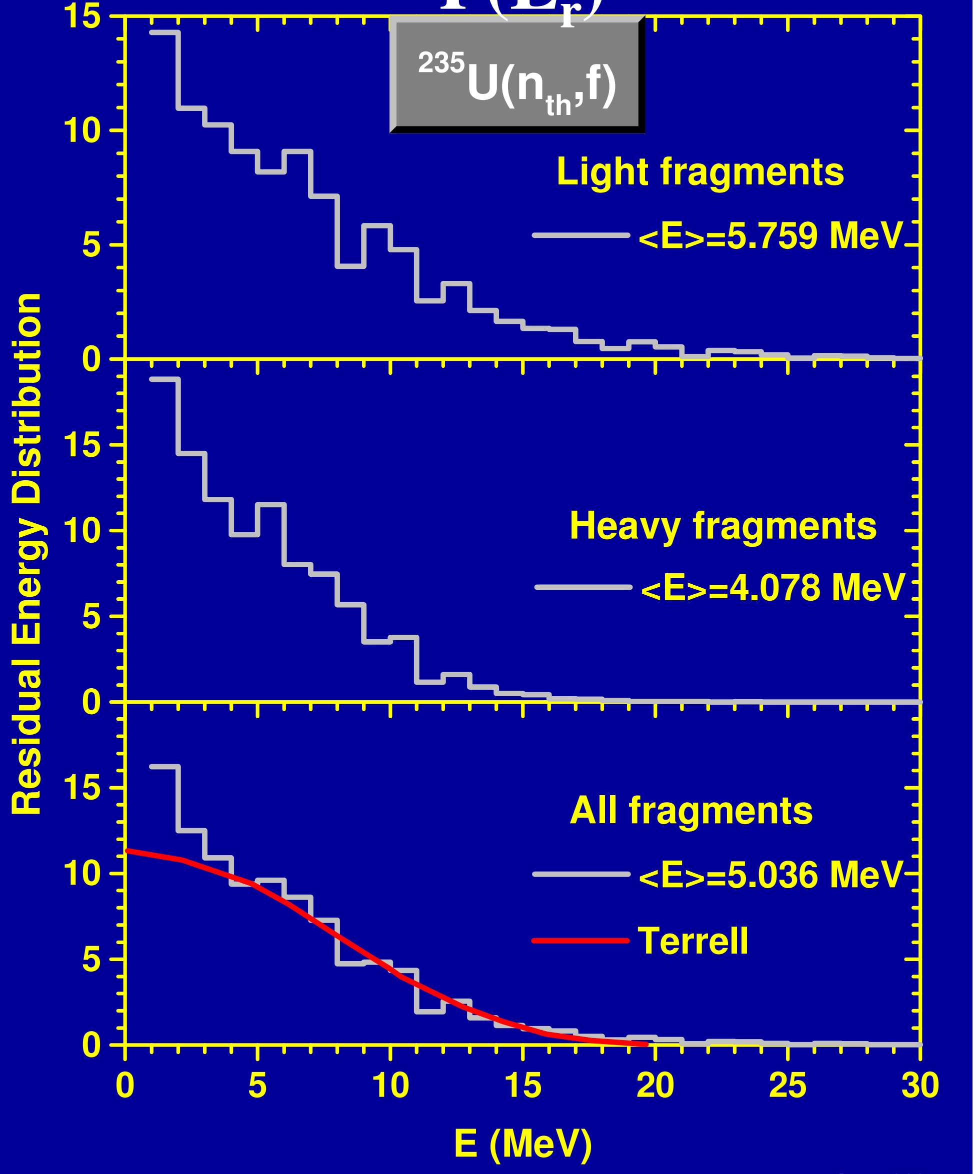
Example of distributions for all sequences

(i.e. sum of distributions following the emission of each neutron)

P(T)



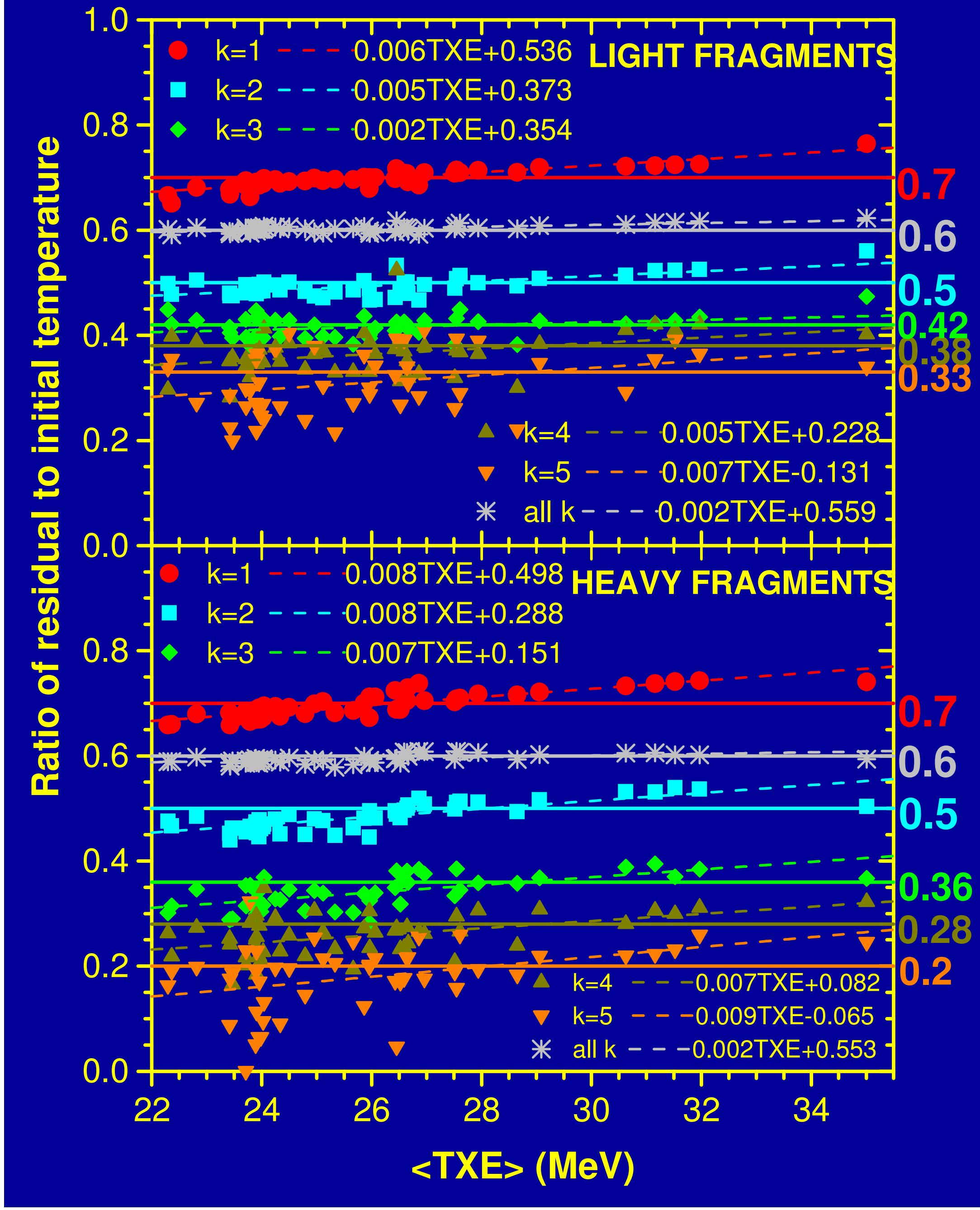
$P(E_r)$



Systematic behaviours of different quantities characterizing the residual fragments and the prompt neutron emission

resulting from sequential emission calculations for 49 fission cases

Ratios of the average residual temperature to the initial temperature



In the case of first two sequences the constant values of the ratios corresponding to LF and HF are equal: r_1=0.7, r_2=0.5

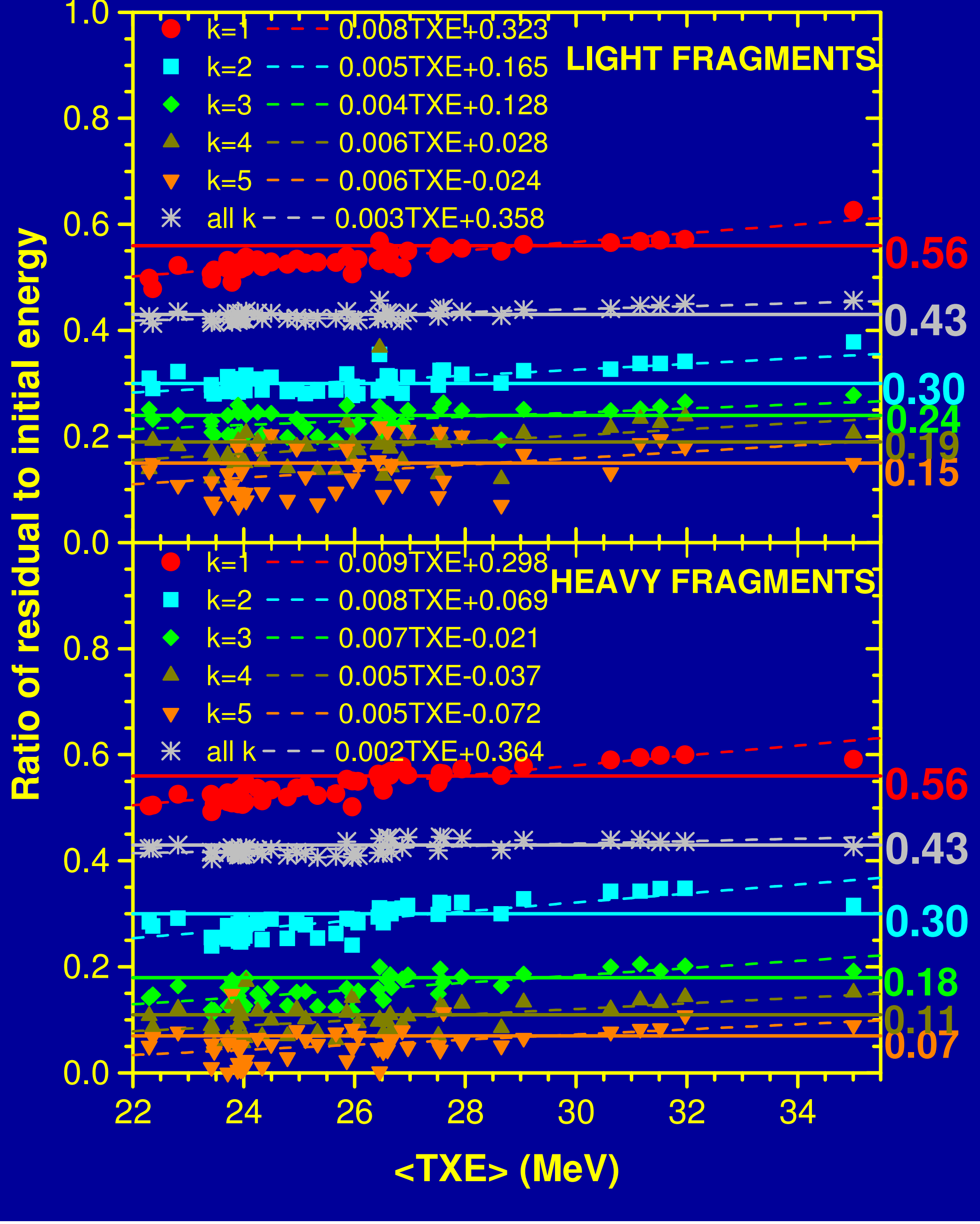
The const.value corresponding to all emission sequences, r_{all}=0.6 is just the mean of the constant values for k=1 and 2.

Because the first two sequences take place for the majority of fragments at a great part of TKE values.

For k > 2 the constant values of the ratios corresponding to LF are higher than those of HF.

Due to the larger difference $\Delta E_k = \langle E_k \rangle_{LF} - \langle E_k \rangle_{HF}$ (of about 1.5 - 2 MeV) for k > 2 compared to k=1 and 2 (less than 1 MeV).

Ratios of the average residual energy to the excit. energy of initial fragm.



In the case of first two sequences, the constant values of the ratios corresponding to LF and HF are equal: $r_1=0.56$, $r_2=0.30$

The const.value corresponding to all emission sequences, $r_{all}=0.43$ is just the mean of the constant values for k=1 and 2.

Because the first two sequences take place for the majority of fragments at a great part of TKE values.

For k > 2 the constant values of the ratios corresponding to LF are higher than those of HF. Due to the larger difference $\Delta E_k = \langle E_k \rangle_{LF} - \langle E_k \rangle_{HF}$ (of about 1.5 - 2 MeV) for k > 2 compared to k=1 and 2 (less than 1 MeV).

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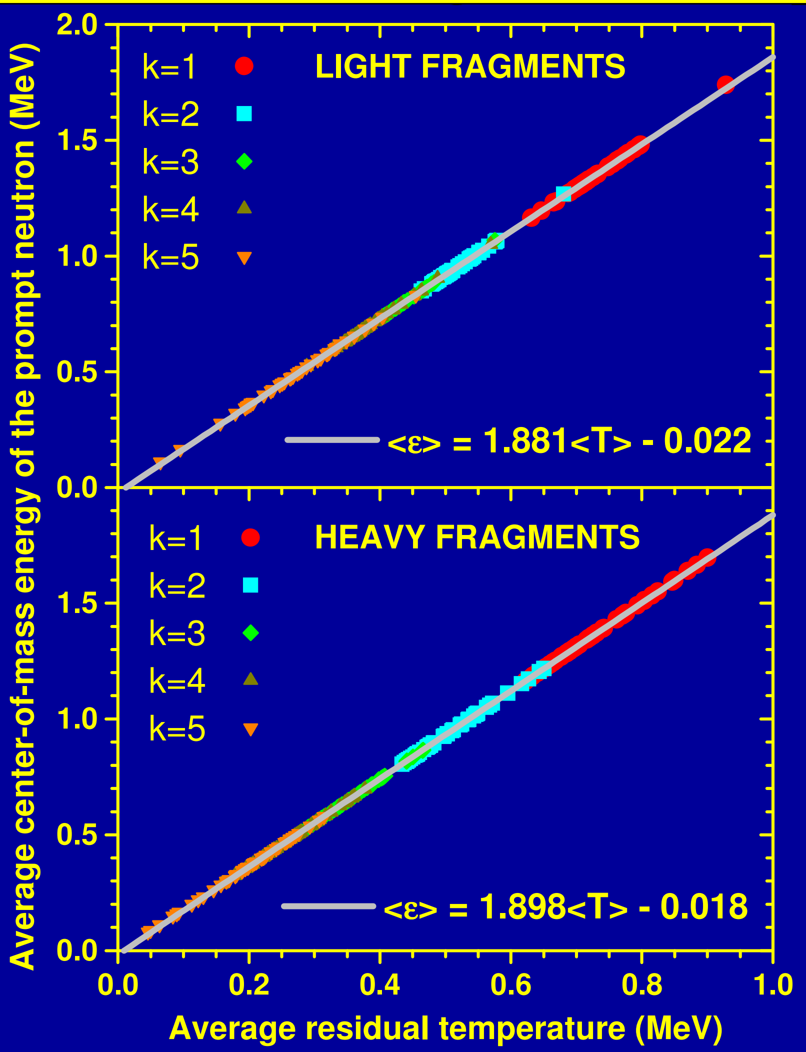
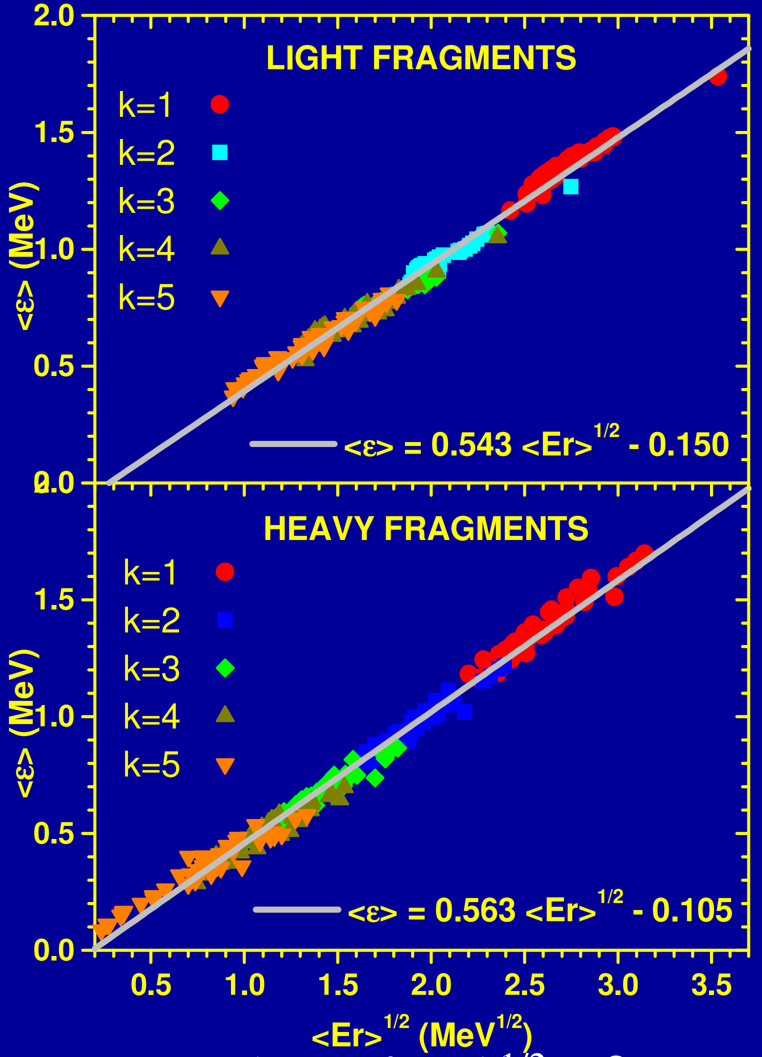
Average center-of-mass energy of prompt neutrons $\langle\varepsilon\rangle_k$ as a function of average residual temperature $\langle T_k\rangle$ and residual energy $\langle Er_k\rangle^{1/2}$

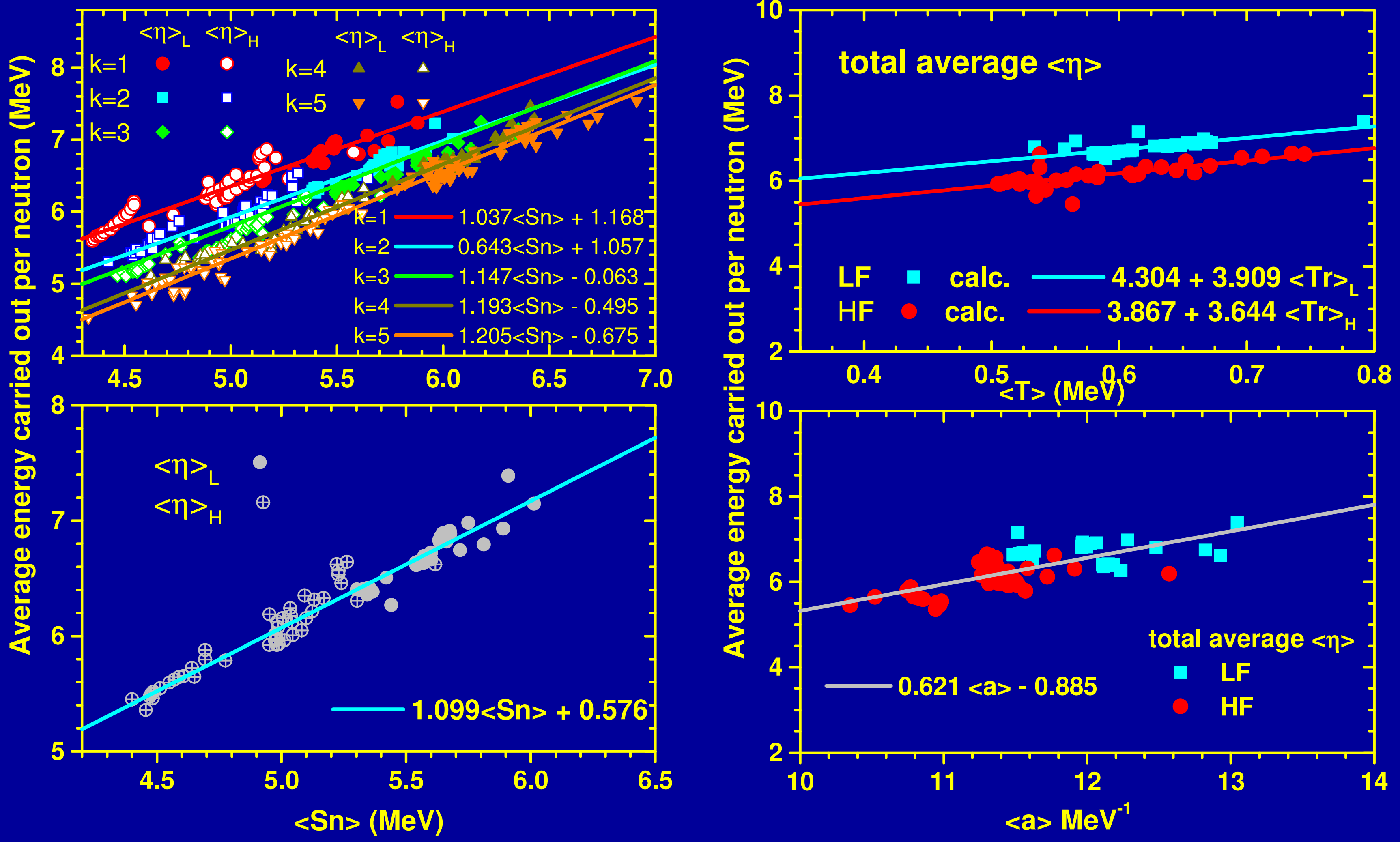




From $\langle\varepsilon\rangle = \alpha\,\langle T\rangle + \beta$ and $\langle E_r\rangle = \langle a\rangle\langle T\rangle^2$ → $\langle\varepsilon\rangle = \alpha\,(\langle Er\rangle/\langle a\rangle)^{1/2} + \beta$

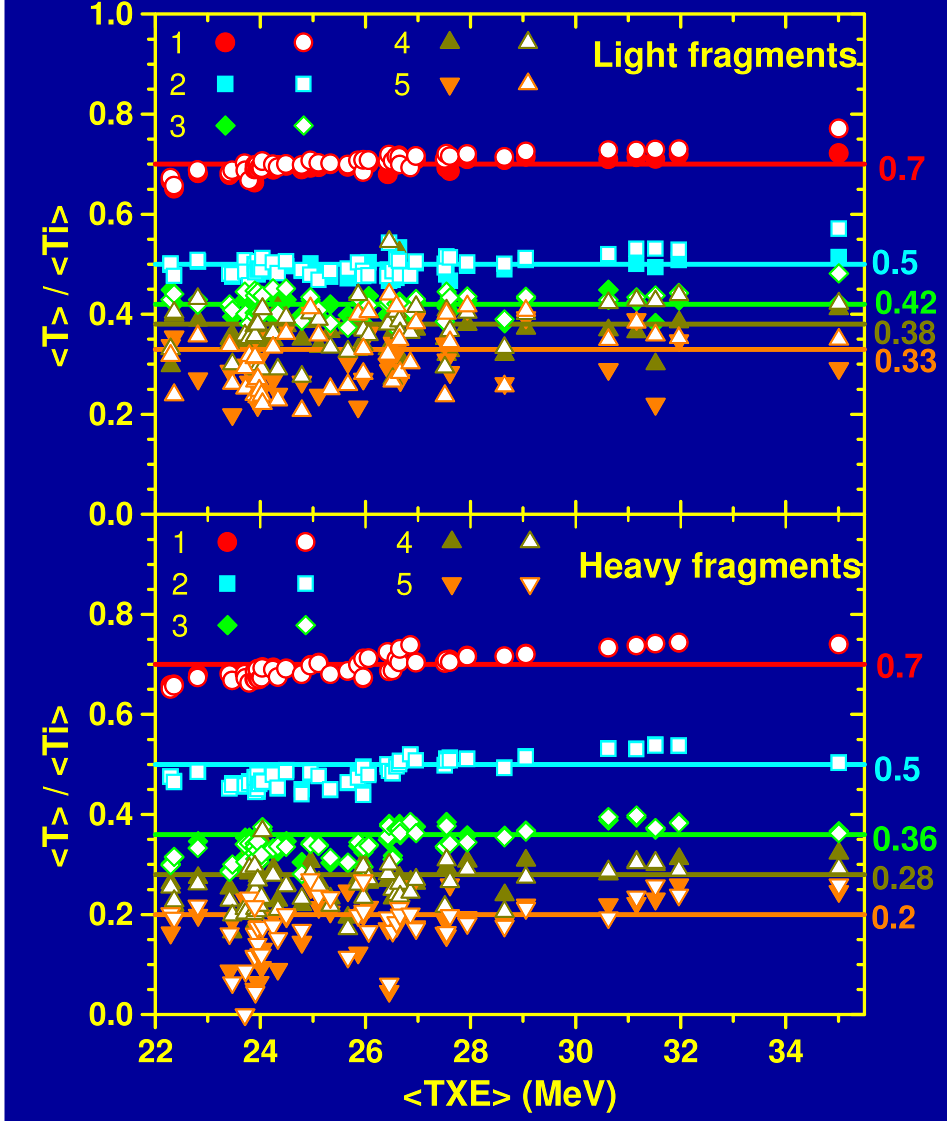
Using the slopes form these figures → global values of the average level density param. of the light and heavy fragment groups: $\langle a\rangle_L = 12\ \mathrm{MeV}^{-1}$, $\langle a\rangle_H = 11.37\ \mathrm{MeV}^{-1}$

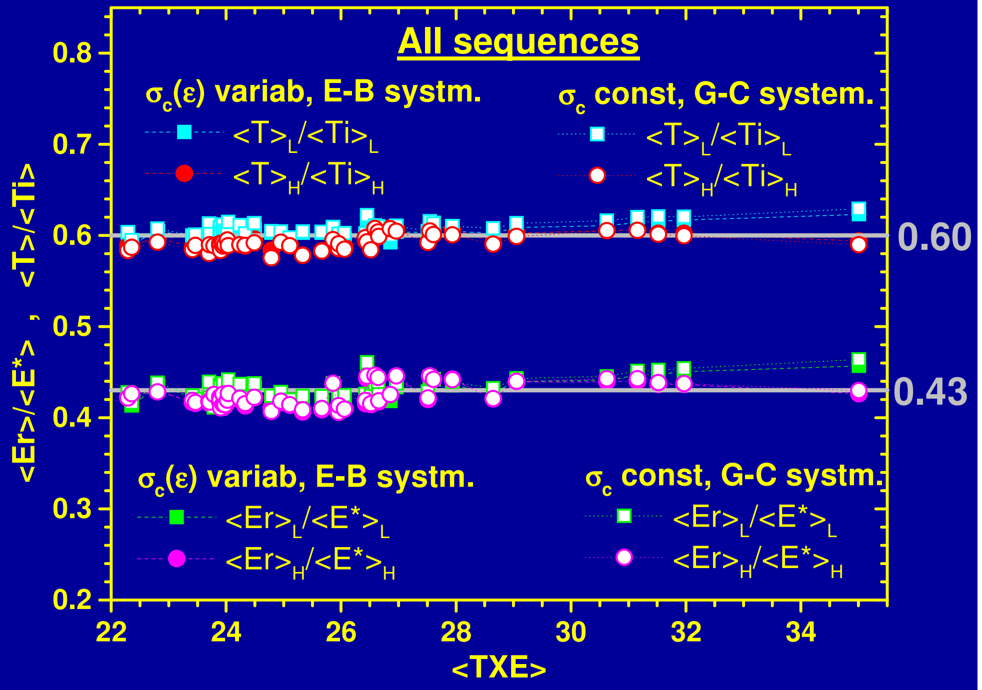
Energy carried away per neutron $\eta_k = <\varepsilon>_k + Sn^{(k-1)}$



- η_k is higher for LF than for HF because $<Sn^{(k-1)}>$ of LF are higher than of HF
- for both LF and HF $\eta_1 > \eta_2 > \eta_3$ … due to the more pronounced decrease of $<\varepsilon>_k$ with increasing of k compared to the concomitantly slowly decrease of <Sn> with the increase of the sequence number k.

Other prescriptions for $\sigma_c(\varepsilon)$ and level density parameters do not change the results. → Here examples for $\sigma_c(\varepsilon)$=constant and level dens. param. provided by the Gilbert-Cameron systematic for spherical nuclei. These prescriptions differ considerably from the ones previously employed.





Full symbols: previous prescriptions (analytical expression $\sigma_c(\varepsilon)$ and E-B 2009 systm. for BSFG)

Open symbols: present prescriptions (constant $\sigma_c(\varepsilon)$ and G-C systematic)

Applications of these systematic behaviours

Sequential emission included into the Los Alamos model

The global treatment of sequential emission using a distribution P(T) ➔ up to now:

$$P(T)=\begin{cases}2T/T_{\max} & T\le T_{\max}\\ 0 & T>T_{\max}\end{cases}$$

$$T_{\max}=<T_i>_{equiv}=\sqrt{<TXE>/<a>}$$ $$<a>=A_0/C$$

Initial LA model of ***Madland and Nix , NSE 1982***

$$T_{\max}=<T_i>_{L,H}=\sqrt{<E^*_{L,H}>/<a_{L,H}>}$$

LA of ***Madland and Kahler, NPA 2017***

$$T_{\max}=T_i(A,Z,TKE)=\sqrt{<E^*(A,Z,TKE>/<a(A,Z,TKE>}$$

PbP model
Tudora et al.

$$<T>/<T_i>=0.6 \Rightarrow T_{\max}=(3/2)<T>=0.9<T_i>$$

Tudora et al. EPJA 2018

The systematic behaviours of $<T_k>/<T_i>$ allow to define <u>a residual temp. distribution for each emission sequence</u> $P_k(T)$, with the maximum temp.:

$$T^{(k)}_{\max\,L,H}=\frac{3}{2}r_{k\,L,H}<T_i>_{L,H}$$

with $r_{1\,L,H}$=0.7, $r_{2\,L,H}$=0.5, $r_{3\,L}$=0.42, $r_{3\,H}$=0.36, $r_{4\,L}$=0.38, $r_{4\,H}$=0.28 etc. resulting from the present systematic of $<T_k>/<T_i>$

Center of mass energy spectrum of each prompt neutron "k" successively emitted from the light or heavy fragment of the most probable fragmentation

$$\Phi_k(\varepsilon) = \int_0^{T\max^{(k)}} \varphi_k(\varepsilon,T)\,P_k(T) = \varepsilon\,\sigma_c^{(k)}(\varepsilon) \int_0^{T\max^{(k)}} K_k(T)\,P_k(T)\exp(-\varepsilon/T)\,dT$$

$$K_k(T) = \left(\int_0^{\infty} \varepsilon\,\sigma_c^{(k)}(\varepsilon)\exp(-\varepsilon/T)\,d\varepsilon \right)^{-1}$$

PFNS in the laboratory frame of each neutron emitted from LF or HF

$$N_k(E) = \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \frac{\Phi_k(\varepsilon)\,d\varepsilon}{4\sqrt{E_f\,\varepsilon}}$$

Prompt neutron spectrum corresponding to all emitted neutrons:

$$\Phi_{L,H}(\varepsilon) = \sum_k Pn_k^{(L,H)}\,\Phi_k^{(L,H)}(\varepsilon)$$

$$N_{L,H}(E) = \sum_k Pn_k^{(L,H)}\,N_k^{(L,H)}(E)$$

Input parameters of the LA model (without or with sequential emission)

different prescriptions can be used regarding the following :

a) **$\sigma_c(\varepsilon)$**: constant or an analytical expression (depending on the mass number and the s-wave neutron strength function of the nucleus {Z, A-k+1} or provided by optical model calculations with phenomenological potentials adequate for nuclei appearing as FF

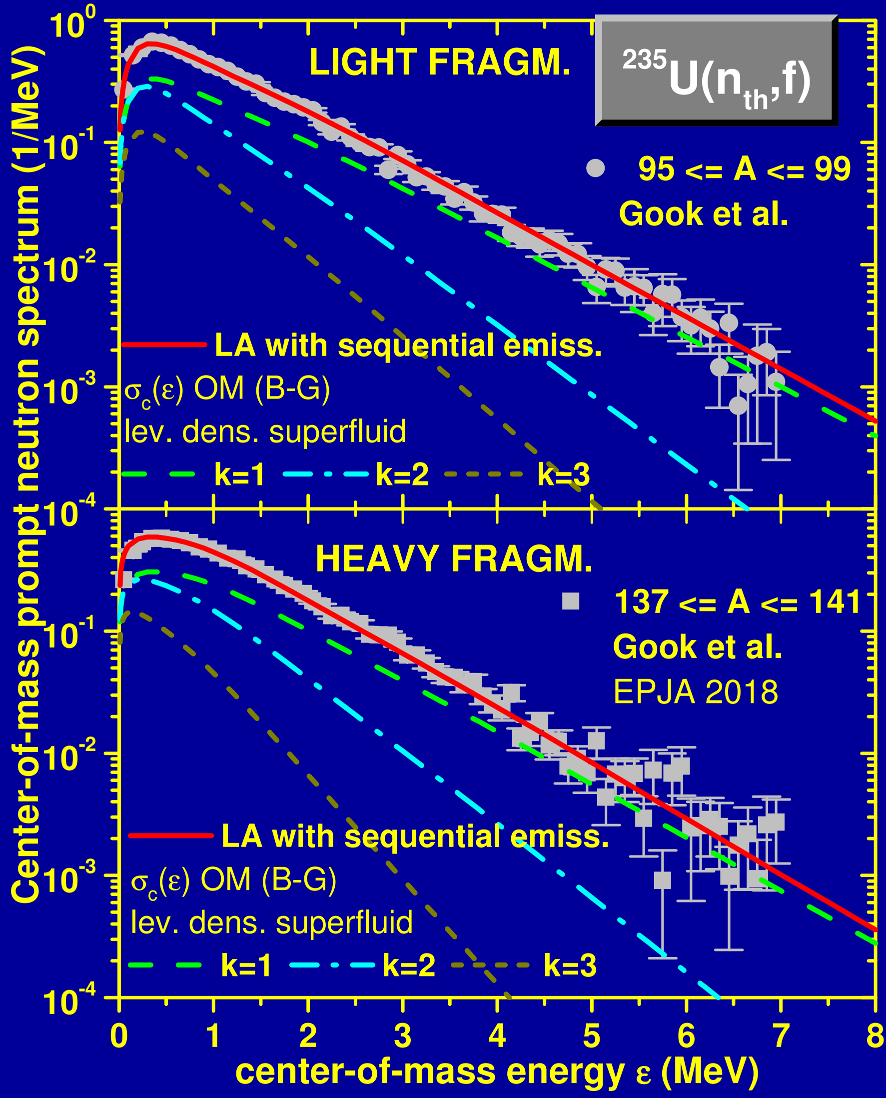
b) **TXE partition**: e.g. by modeling at scission (PbP), the procedure of Madland and Kahler, the method of FIFRELIN (intrinsic energy partition according.to a temp.ratio RT)), FREYA (adjustable param.”x” for LF to mach the experiment), GEF (intrinsic energy partition according to the sorting mechanism) etc.

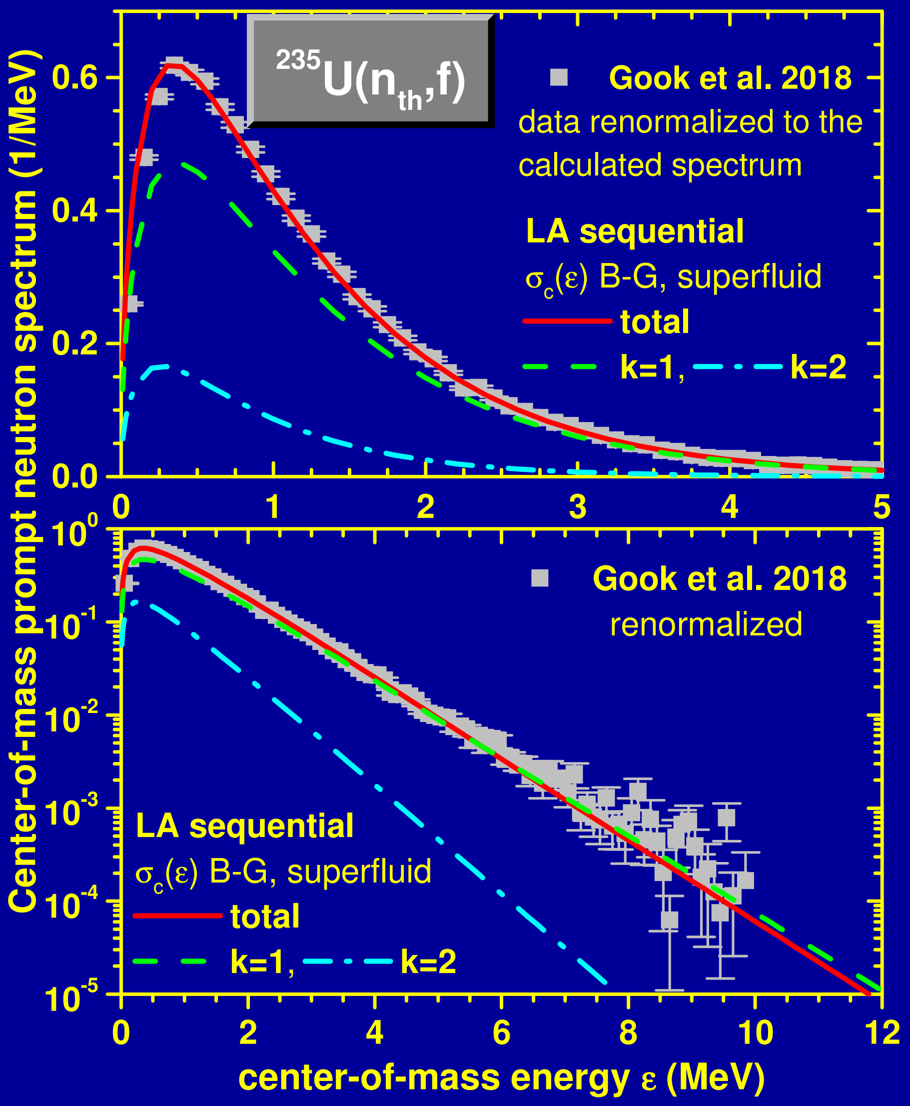
c) **level density parameters** of fragments: either energy-dependent (super-fluid with different shell corrections and parameterizations of the dumping and asymptotic lev. dens. param.) or non-energy dependent (e.g. systematics of EB-2009 for BSFG, G-C etc.)

The prescriptions used in the present calculations (with examples given in the next slides) are marked with a blue pen.

Example of results of the LA model with sequential emission

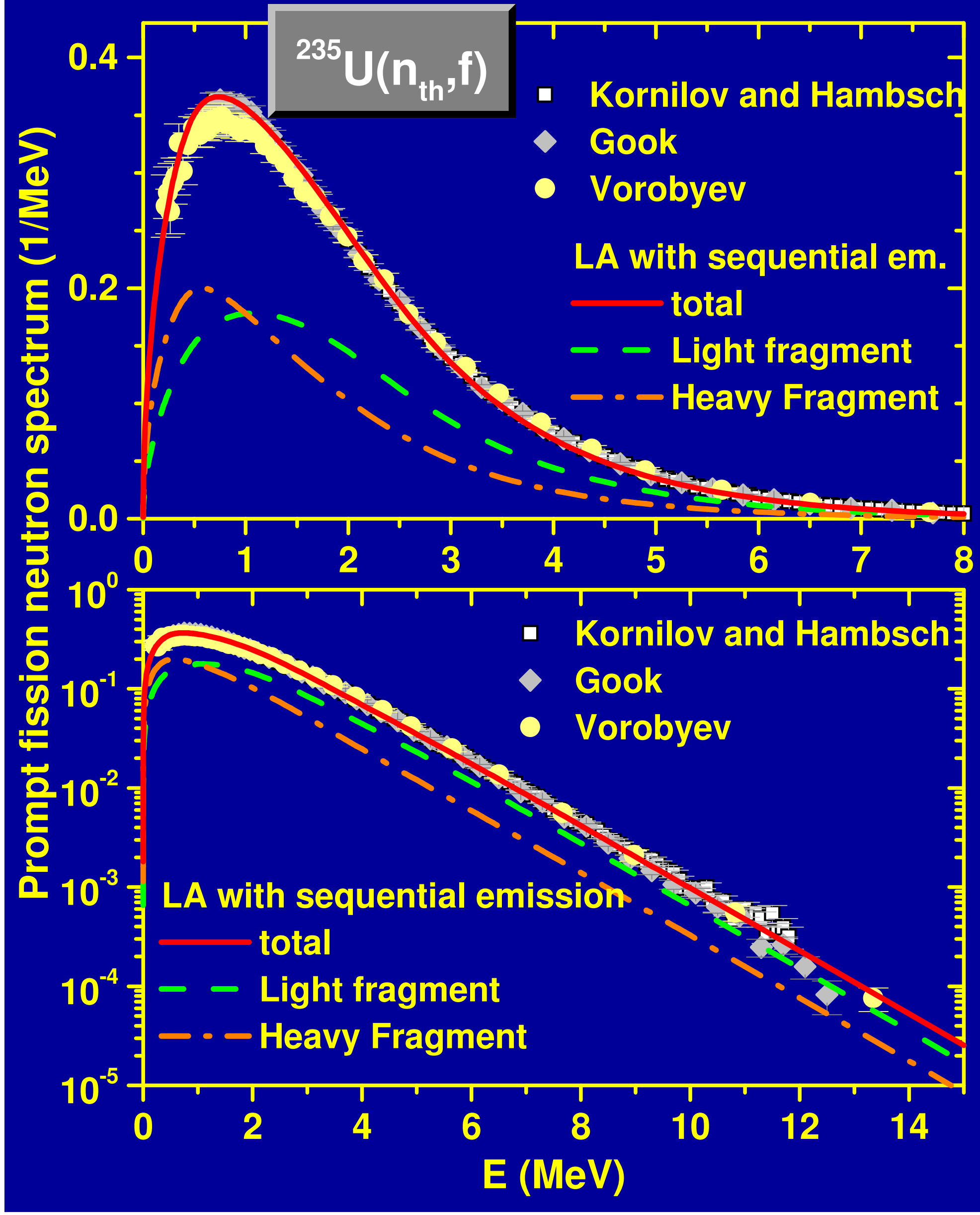
Center-of-mass energy spec. separately for LF and HF (left) and total (right)

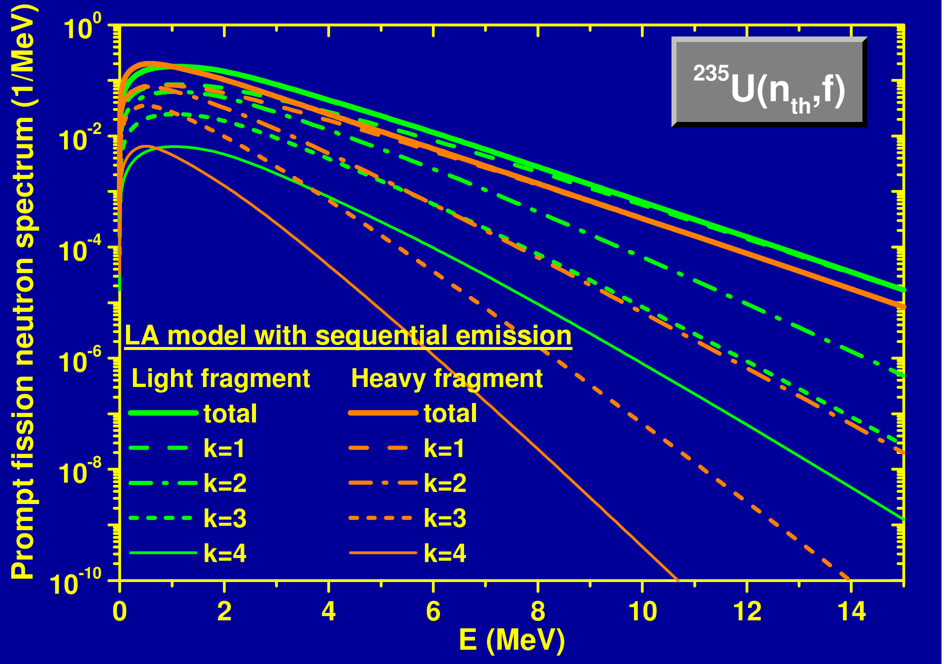




Example of results of the LA model with sequential emission

Prompt neutron spectrum in the laboratory frame





The summed contributions of PFNS for k=1, 2, 3, 4 illustrated above by solid lines (green - LF and orange – HF) are plotted in the left part (using the same colours).

Another application:

The systematic behaviours can be used in order to obtain indicative values of different prompt emission quantities in the absence of any prompt emission model.

If the average temperatures of initial fragments $<T_i>_{L,H}$ or $<T_i>_{equiv}$ are known then $<\varepsilon>$ can be obtained from the linear behaviour $<\varepsilon>_k = \alpha<T_k> + \beta$ using the values of $<T_k>$ from the systematic of temperature ratios $<T_k>/<T_i>$.

Example:

^{252}Cf(SF): $<TXE>=35.01$ MeV, $<E^*>_L=19.97$ MeV, $<E^*>_H=15.04$ MeV

a) Using $<T_i>_{equiv}$ based on $<TXE>$ and $<a> = 252/11$ MeV^{-1}
$<\varepsilon> = 1.38$ MeV is obtained (in agreement with exp.data Göök et al.)

b) Using $<T_i>_{L,H}$ based on $<E^*>_{L,H}$ and level density parameters of the super-fluid model $<a_L>=13.55$ MeV^{-1}, $<a_H>=12.76$ MeV^{-1} the following values are obtained $<\varepsilon>_L=1.43$ MeV, $<\varepsilon>_H=1.27$ MeV and $<\varepsilon>=1.36$ MeV which deviate only with 0.7% from the result of Madland and Kahler (NPA 2017).

Conclusions

The deterministic treatment of sequential emission applied to 49 fission cases allowed to obtain systematic behaviours and correlations between different average quantities characterizing the initial and residual fragments and the prompt neutron emission.

1. **The ratios $<T>/<T_i>$ and $<E>/<E^*>$ corresponding to all sequences and $<T_k>/<T_i>$ and $<E_k>/<E^*>$ of each emission sequence are almost the same for all fission cases (constant values) irrespective of the prescriptions used for $\sigma_c(\varepsilon)$ and the level density param. of initial and residual fragments.**

2. **$<T>/<T_i> = 0.6$ → new P(T)** ***(Tudora et al. EPJA 2018)***

$<T_k>/<T_i> = r_k$ (e.g. r_1=0.7, r_2=0.5 for LF, HF, r_3=0.42 (LF), 0.36 (HF) etc.) allow to define $P_k(T)$ with $T_{max}^{(k)} = (3/2)\, r_k <T_i>$ having as application the inclusion of sequential emission into the Los Alamos model.

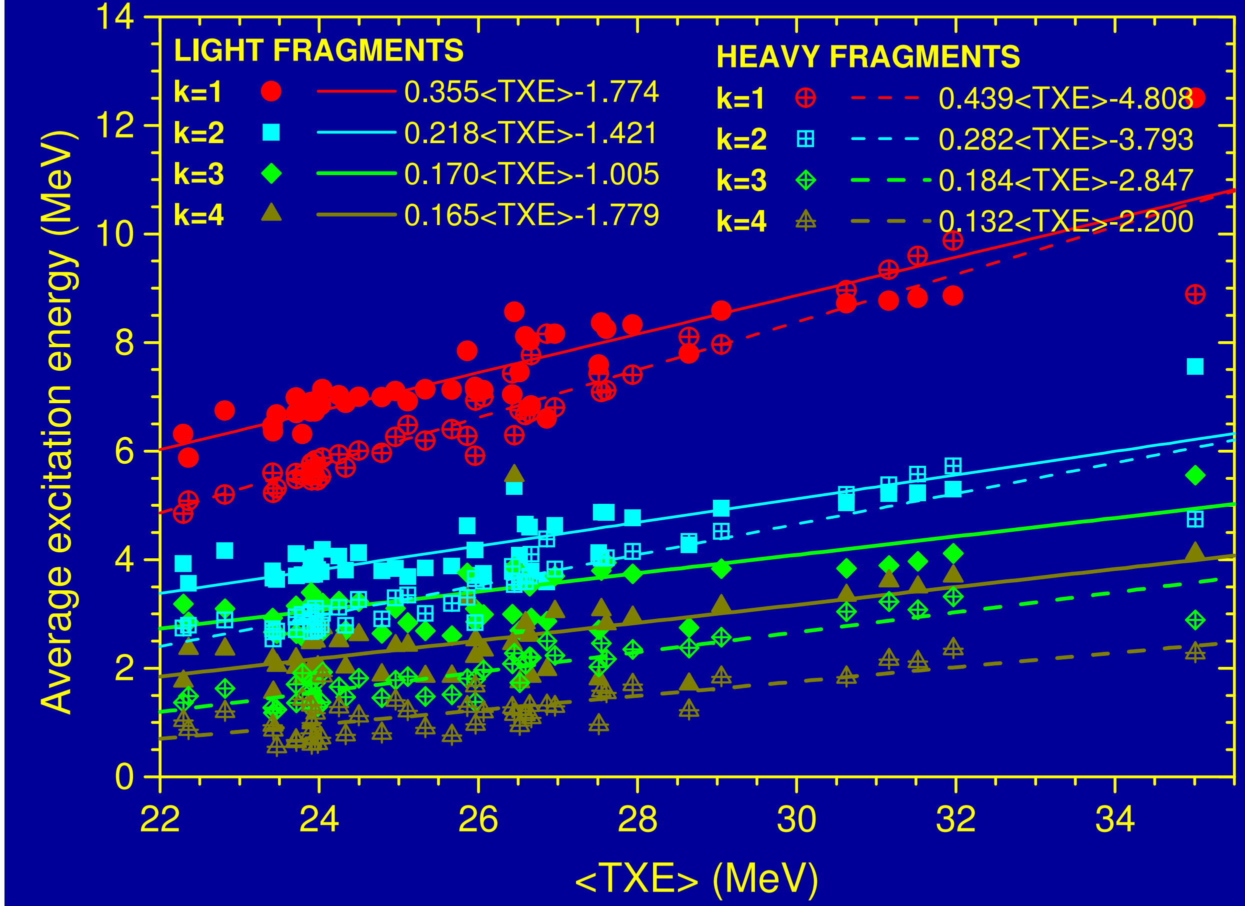
3. **The constant ratios $<T>/<T_i>$ and the linear behaviour of $<\varepsilon>_{L,H}$ as a function of $<T>_{L,H}$ allow to obtain indicative values of different average prompt emission quantities in the absence of any prompt emission model..**

4. **Almost linear dependences of the average energy carried away by each emitted neutron ($<\eta>_k$) on $<Sn>_{k-1}$, on $<T>_k$ and $<a>_k$ are established, too.**

Thanks for your attention

For the emission sequences with k > 2 the residual temperature and energy ratios corresponding to LF are higher than those of HF → due to the magnitude of the difference between the residual energies of LF and HF: $\Delta E_k = \langle E_k \rangle_{LF} - \langle E_k \rangle_{HF}$

For k > 2 these differences are higher (1.5 - 2 MeV) compared to ΔE_k for k=1 and 2 (less than 1 MeV)



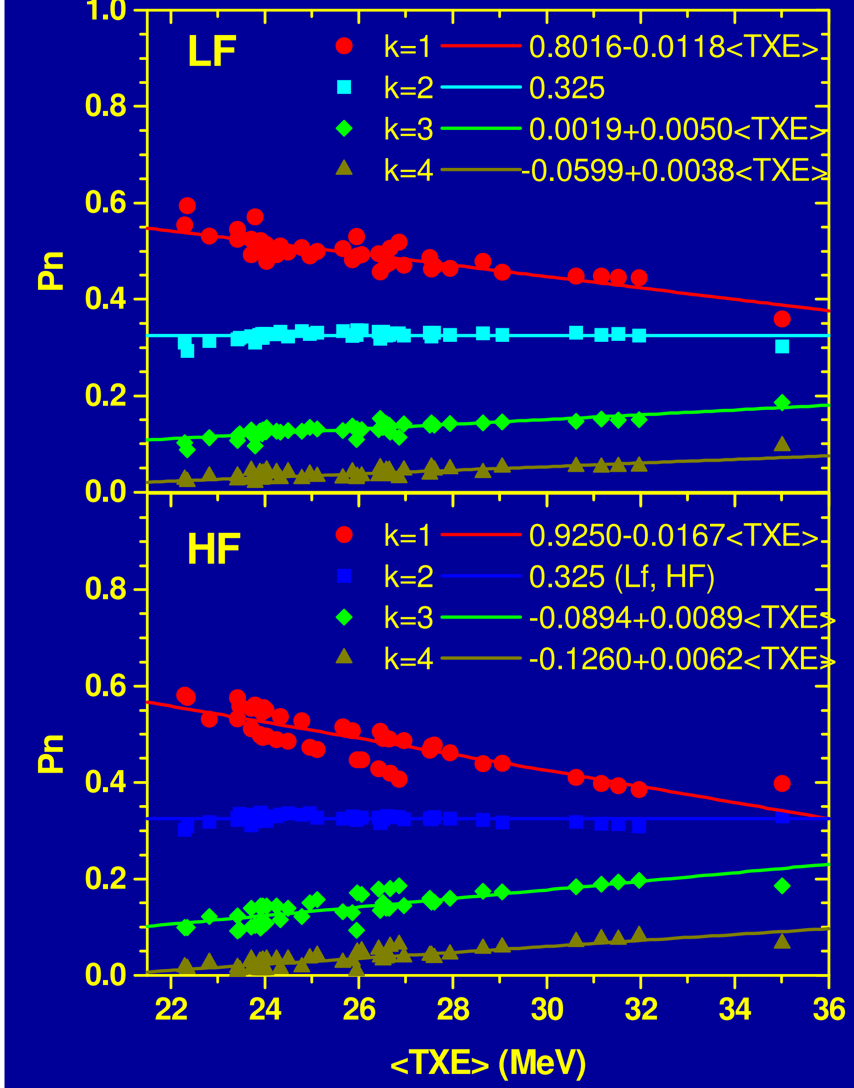
$\langle T_k \rangle / \langle T_i \rangle$ depends on the root-square of $\langle E_k \rangle / \langle E^* \rangle$ and $\langle a_i \rangle / \langle a_k \rangle$

The lev. dens. ratios are close to 1, e.g.

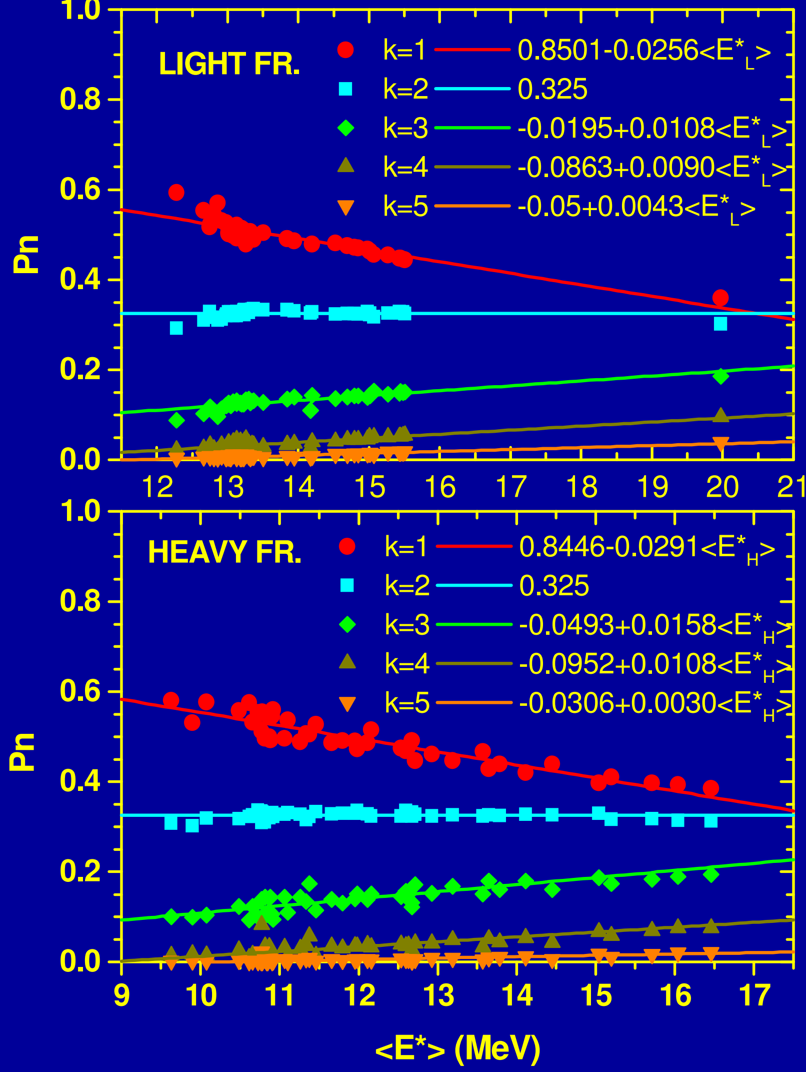
0.99 (LF) 0.98 (HF) k=2
0.96 (LF) 0.93 (HF) k=3
0.94 (LF) 0.88 (HF) k=4

Probability for emission of each prompt neutron Pn_k

Pn_k as a function of <TXE>

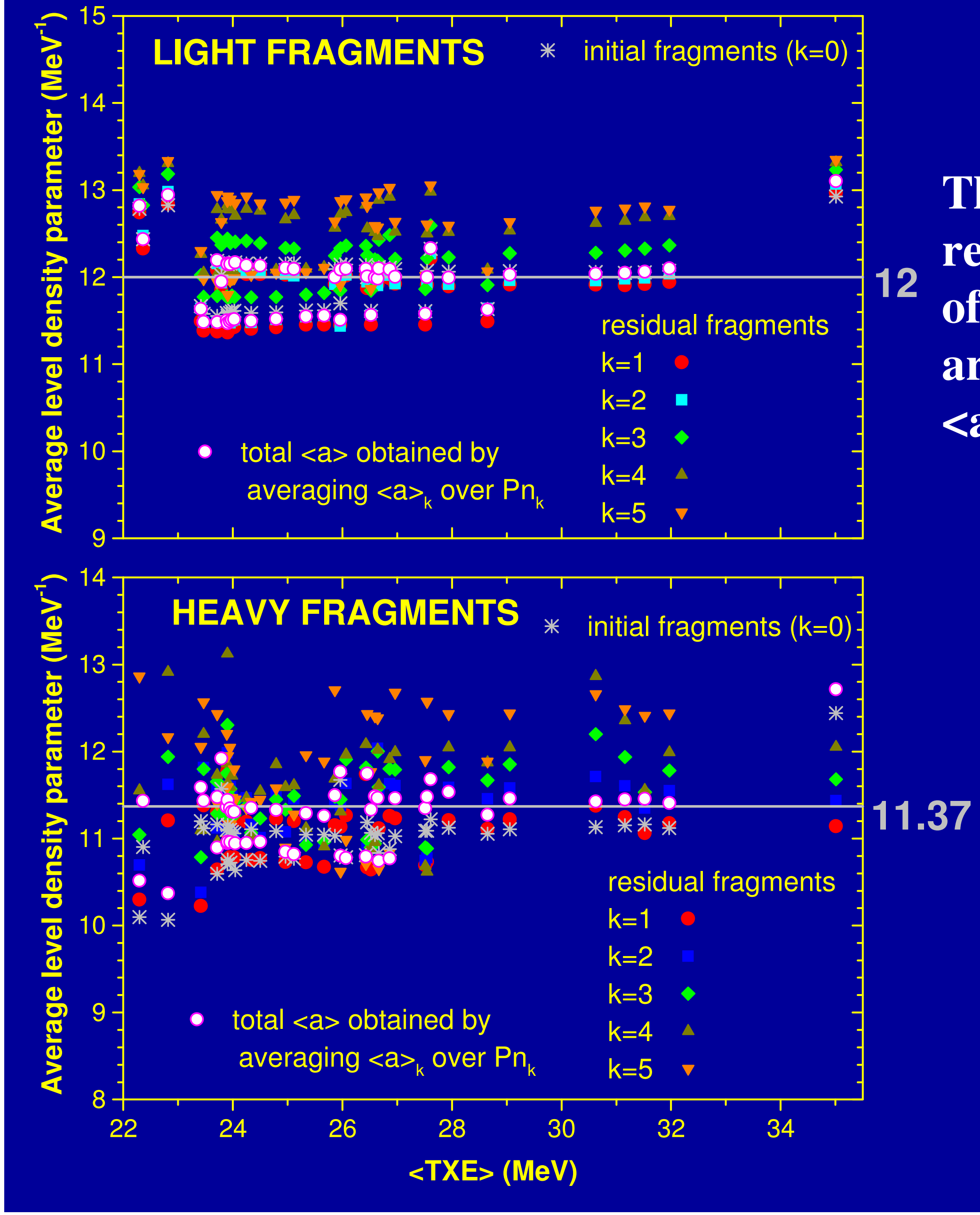


Pn_k as a function of <E*>



Pn_k not confounded with the distribution $P(\nu)$ = probab for emission of 1, 2, 3 … neutrons

Average level density parameters of the initial and residual fragments



The global values of <a> (horizontal lines) resulting from the systematic behaviour of $\langle\varepsilon\rangle_k$ as a func. of $\langle T\rangle_k$ and $\langle Er_k\rangle^{1/2}$ are in agreement with the total average <a> (magenta open circles).

The fact that <a> for **k=1** (red) and **k=2** (blue) **are close** to the **total** <a> (magenta open circles) is not surprising because the first two emission sequences take place for the majority of fragments at a great part of TKE values.

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