I. Preliminary results of a detailed validation of the PbP model of prompt emission

Multi-parametric matrices compared with recent experimental data

II. Preliminary results of a detailed calculation taking into account the successive emission of each prompt neutron (sequential emission)

To obtain a new parameterization of the residual temperature distribution P(T)

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- The first and most important validation of almost all models, including the models of prompt emission, is based on the comparison of model results with existing experimental data.

- The majority of prompt emission models (e.g. PbP, FIFRELIN, CGMF, FREYA) use experimental fission fragment distributions as input data in order to provide different average quantities, i.e. as a function of A, as a function of TKE, as a function of Z and total average quantities. These average quantities are compared with existing experimental data.

The primary results of the **PbP model** are the **multi-parametric matrices** of many quantities referring to fission fragments and prompt emission,

generically labeled as **q(A,Z,TKE)**

e.g. E*(A,Z,TKE), a(A,Z,TKE), Sn(A,Z,TKE), ν(A,Z,TKE), Eγ(A,Z,TKE), <ε>(A,Z,TKE), Φ(A,Z,TKE, ε), N(A,Z,TKE, E) etc.

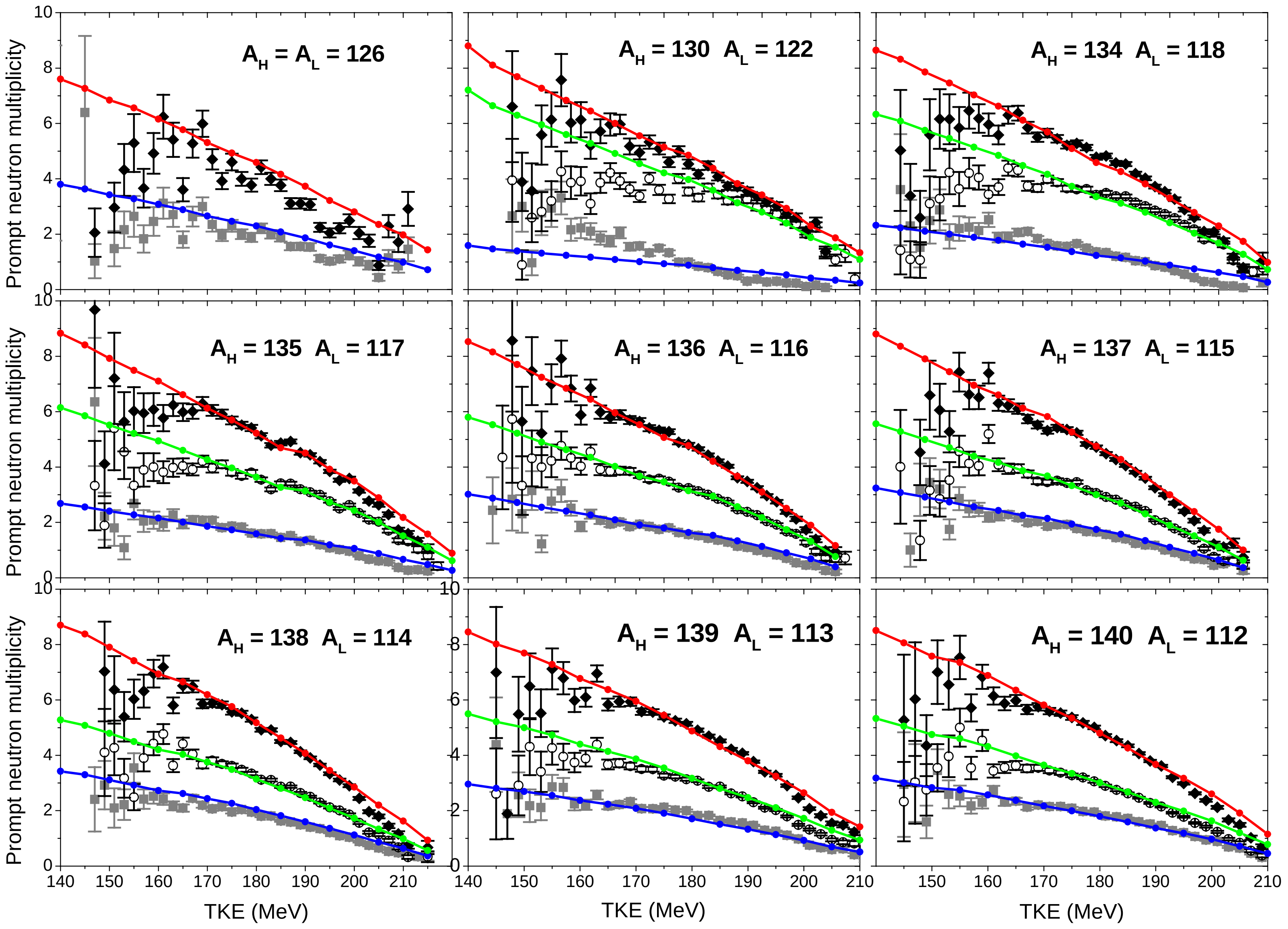
These multi-parametric matrices do not depend on fragment distributions. For this reason

the comparison of such quantities (as a func. of A, Z, TKE) with existing experimental data is the most important, validating the model itself.

The recent ν(A,TKE) data of ^{252}Cf(SF) measured by *Göök et al. ,PRC 2014* offer the possibility of a detailed validation of the PbP model itself

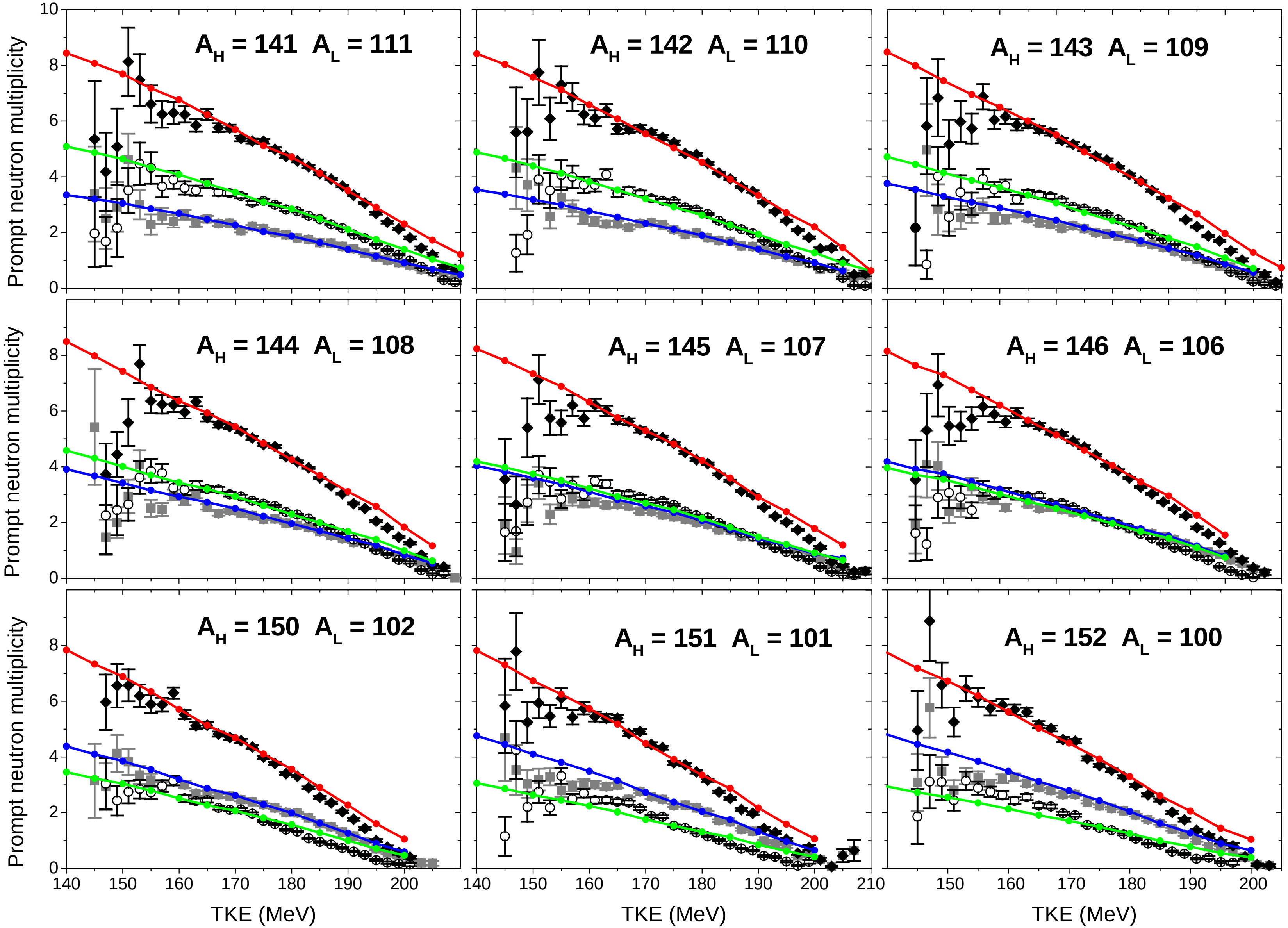
^{252}Cf(SF)

ν(A,TKE) matrix: PbP calculation and exp. data of Göök et al., 2004



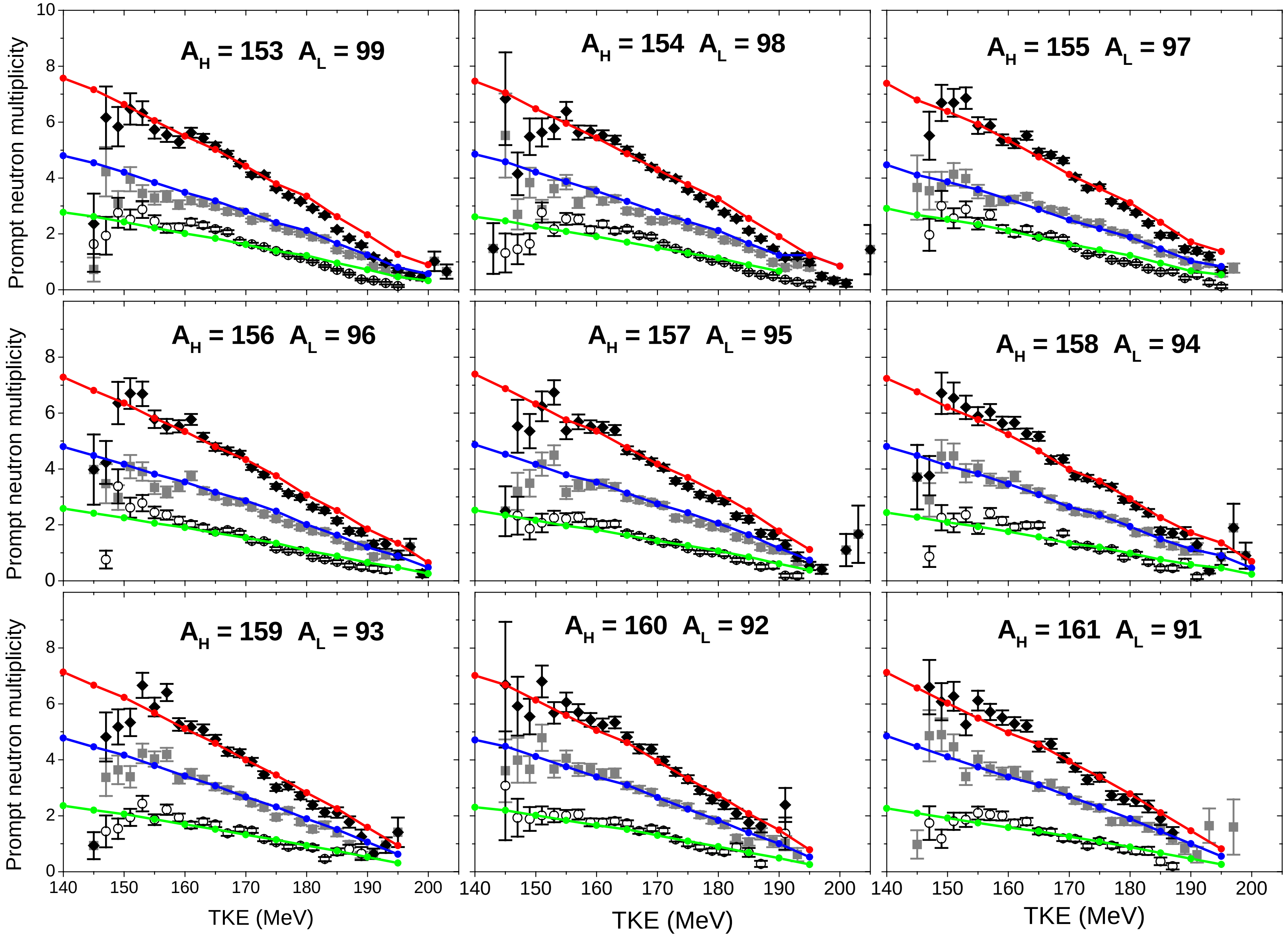
^{252}Cf(SF)

ν(A,TKE) matrix: PbP calculation and exp. data of Göök et al.



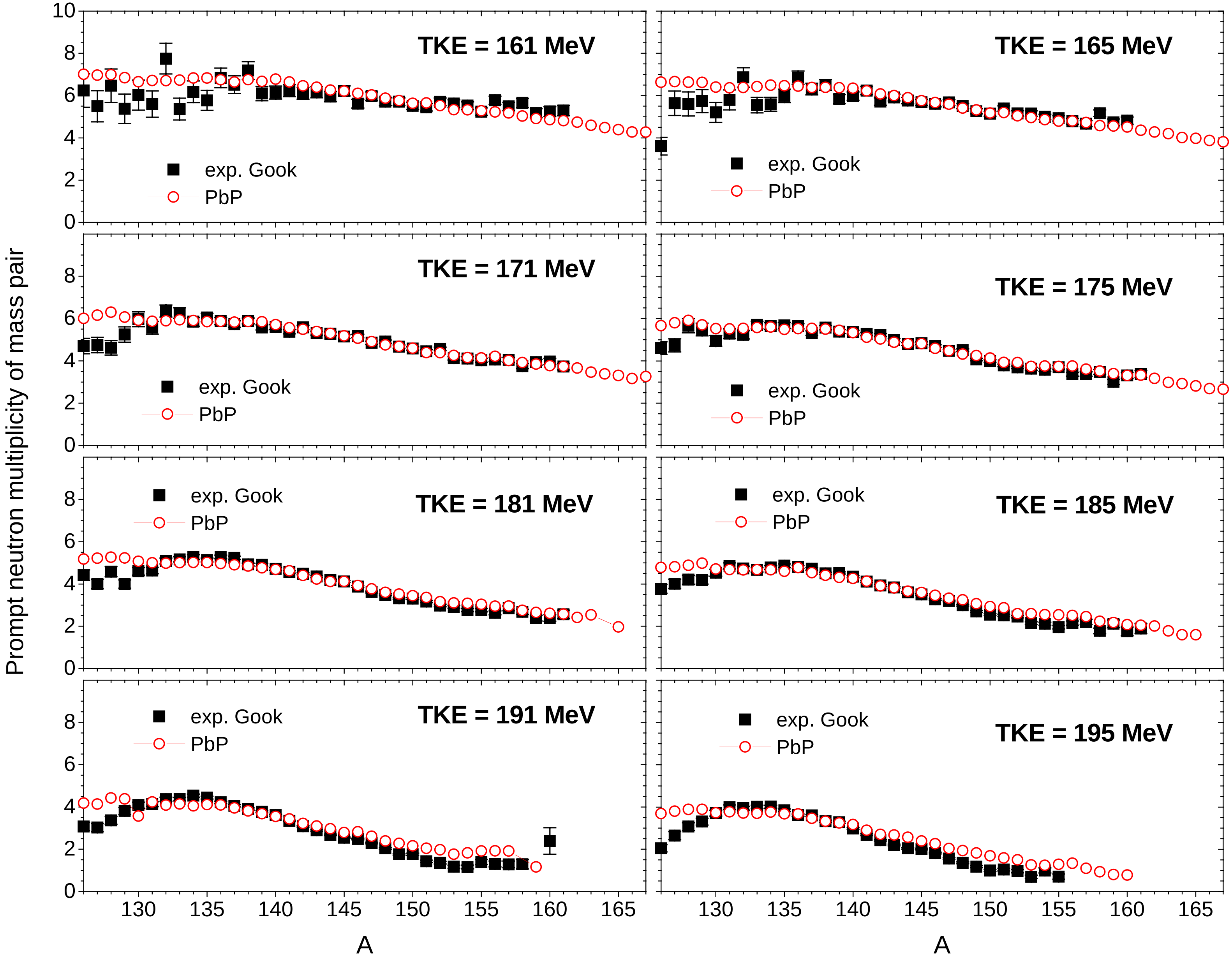
^{252}Cf(SF)

ν(A,TKE) matrix: PbP calculation and exp. data of Göök et al.

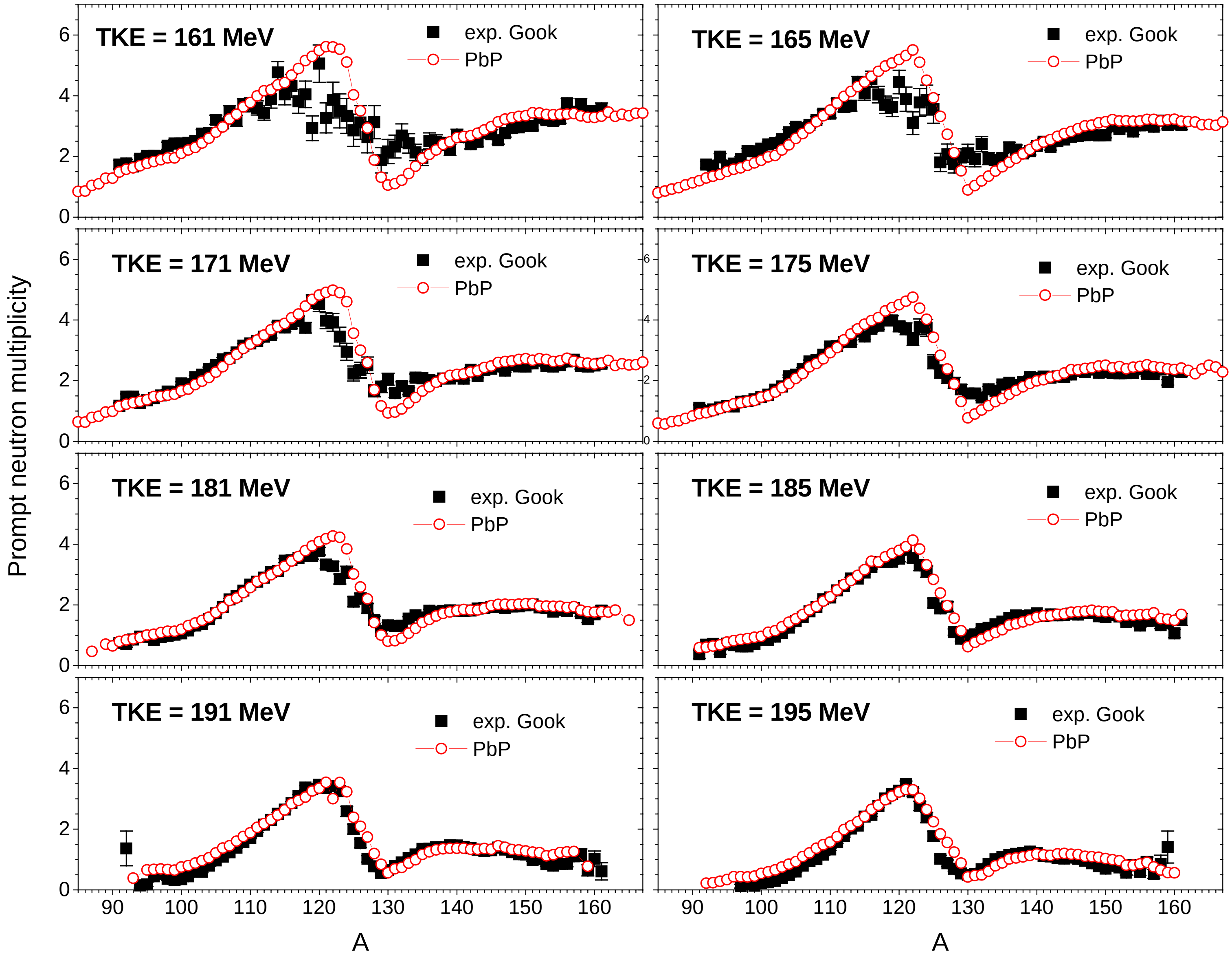


^{252}Cf(SF)

$\nu_{pair}(A_H, TKE)$ **matrix**: PbP calculation and exp. data of Göök et al.

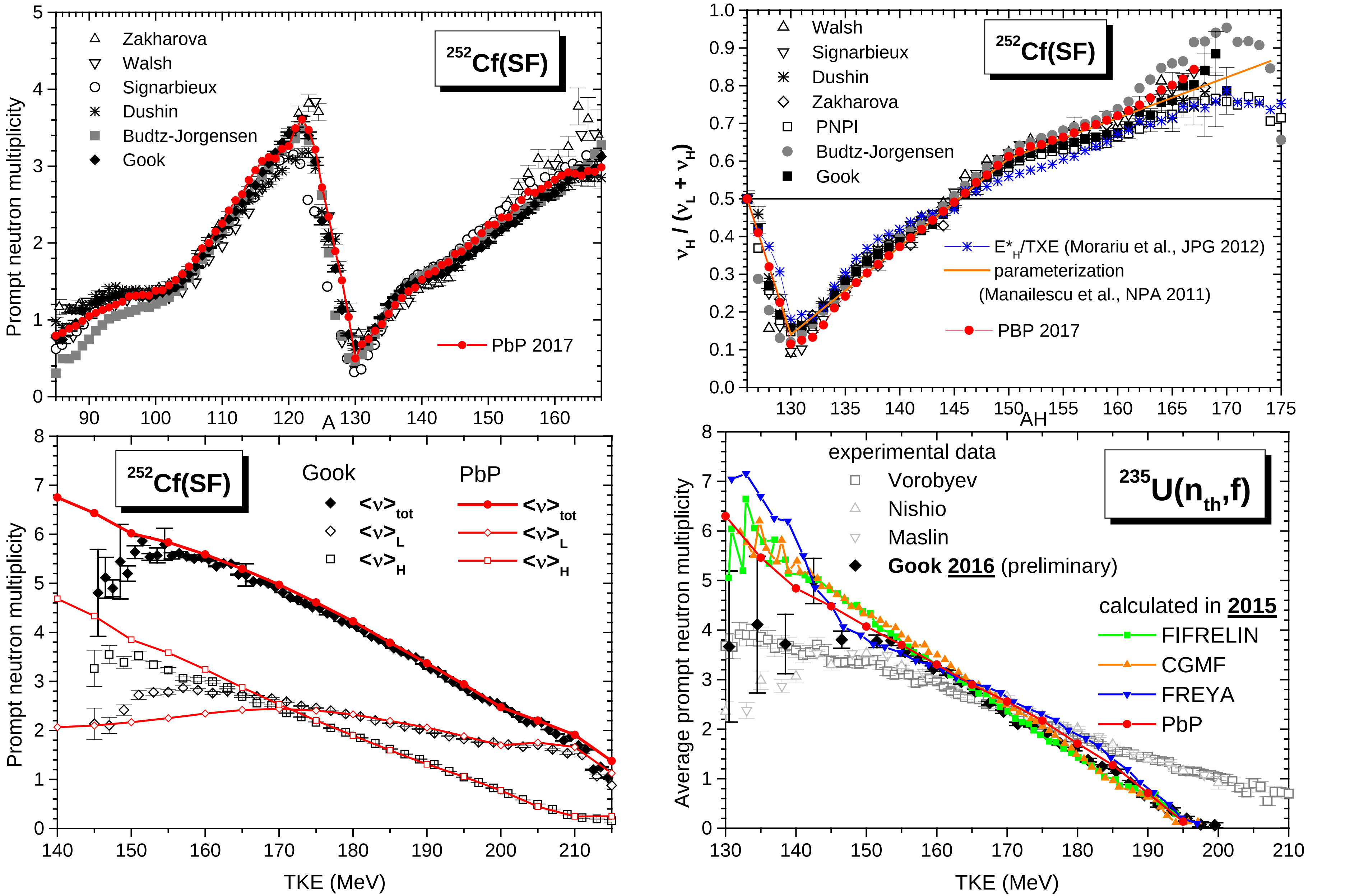


^{252}Cf(SF) **ν(A,TKE) matrix**: PbP calculation and exp. data of Göök et al.



Second validation – comparison of average quantities with experim. data

these quantities depend on fragment distributions (exp.Y(A,TKE) Göök are used)



II. PRELIMINARY RESULTS of a detailed calculation following the successive emission of each prompt neutron

This sequential emission calculation provides the distributions of residual temperatures P(T) allowing to obtain a new parameterization of P(T) as a function of the temperature of initial fragments

- **short description of the modeling → equations giving the residual Tr and Er following the emission of each neutron**
- **distributions of Tr, Er, <ε> etc. following the emission of each neutron**
- **sum of the distributions of Tr, Er, etc. following the successive emission of all neutrons (from HF, LF and all fragments)**
- **different quantities corresponding to the emission of each neutron and to the successive emission of all neutrons, both as a function of A and TKE of initial fragments**
- **validation by comparison with experim. data ν(A), ν(TKE), <ε>(A) etc. and with results of other prompt emission models (PbP, GEF, FIFRELIN, etc.)**
- **a new form of the residual P(T) for HF, LF and all FF (parameterized as a function of T of the initial fragment) – to be used in the the PbP model and also in the LA model**

The evaporation spectrum of a neutron from a fragment in the center-of-mass frame for a given residual temperature Tr :

$$\varphi(\varepsilon,T_r) = K(T_r)\sigma_c(\varepsilon)\varepsilon\exp(-\varepsilon/T_r) \qquad K(T_r) = \left(\int_0^\infty \varphi(\varepsilon,T_r)\,d\varepsilon\right)^{-1}$$

In the deterministic model PbP and in the LA model the successive emission of neutrons is globally taken into account by by the residual temp. distribution $P(T_r)$

The prompt neutron spectrum in the center-of-mass frame corresponding to a fragment:

$$\Phi(\varepsilon) = \int_0^{T\max} P(T_r)\varphi(\varepsilon,T_r)\,dT_r$$

Detained calculations taking into account the successive neutron emission (sequential emission) allow to obtain the residual temperature distribution following the emission of each neutron (indexed k) as well as other distributions (e.g. of the residual energy, of the average neutron energy in CMS etc.)

The residual temperatures $T_r^{(k)}$ (of the k-th residual nucleus, after the emission of the k-th neutron) is the solution of the following equation:

$$\overline{E_r}^{(k-1)} - S_n^{(k-1)} - <\varepsilon>_k (T_r^{(k)}) = a_k T_r^{(k)2}$$

for the k-th emitted neutron and the k-th residual nucleus

k = 1 $\overline{E_r}^{(0)} = E^*$ **excitation energy of the initial fragment obtained from the TXE partition based on modeling at scission**

Approximations needed to solve the iterative equations of residual temperatures

$$\overline{E_r}^{(k-1)} - S_n^{(k-1)} - <\varepsilon>_k (T_r^{(k)}) = a_k T_r^{(k)2}$$

a) fragment level density in the Fermi-Gas regime with a non-energy dependent level density parameter a_k , e.g.:

- systematic of Egidy-Bucurescu (2009) for the BSFG model

- systematic of Gilbert-Cameron for spherical nuclei

b) an analytical expression of $\sigma_c(\varepsilon)$ approximating $\sigma_c(\varepsilon)$ provided by optical model calculations (with an optical potential parameterization appropriate for nuclei appearing as fission fragments, e.g. Becchetti-Greenlees, Koning-Delaroche)

$$\sigma_c^{(k)}(\varepsilon) = \sigma_0^{(k)}\left(1+\alpha_k/\sqrt{\varepsilon}\right) \rightarrow \quad <\varepsilon>_k (T_r^{(k)}) = \frac{T_r^{(k)}\left(2\sqrt{T_r^{(k)}} + 3\alpha_k\sqrt{\pi}/4\right)}{\left(\sqrt{T_r^{(k)}} + \alpha_k\sqrt{\pi}/2\right)}$$

with $\sigma_0^{(k)}$ and α_k depending on the mass number and the s-wave neutron strength function S_0 of the each nucleus (Z, A-k+1) (with k = 1 to k_{max}(A,Z,TKE))

The residual temperatures $T_r^{(k)}$ are solutions of transcendent equations.

c) $\sigma_c(\varepsilon)$ = constant → $\quad <\varepsilon>_k (T_r^{(k)}) = 2\,T_r^{(k)}$

analytical solutions: $$T_r^{(k)} = \frac{1}{a_k}\left(\sqrt{1+a_k(\overline{E_r}^{(k-1)} - S_n^{(k-1)})} - 1\right)$$

Comparison of <ε> based on an analytical formula of $\sigma_c(\varepsilon)$ with <ε> based on $\sigma_c(\varepsilon)$ from optical model calculations

OM calc.

$$\langle\varepsilon\rangle(T) = \int_0^\infty K(T)\,\varepsilon^2\,\sigma_c(\varepsilon)\exp(-\varepsilon/T)\,d\varepsilon$$

$$K(T) = \left(\int_0^\infty \varepsilon\,\sigma_c(\varepsilon)\exp(-\varepsilon/T)\,d\varepsilon\right)^{-1}$$

$$\sigma_0 = \pi R^2 \qquad R = r_0 A^{1/3}$$

$$\sigma_S(\varepsilon) = \frac{\pi}{k^2}T_0 = \frac{(\pi\hbar)^2}{m\sqrt{\varepsilon}}S_0$$

$$\sigma_c(\varepsilon) = \sigma_0\left(1 + \frac{\alpha}{\sqrt{\varepsilon}}\right)$$

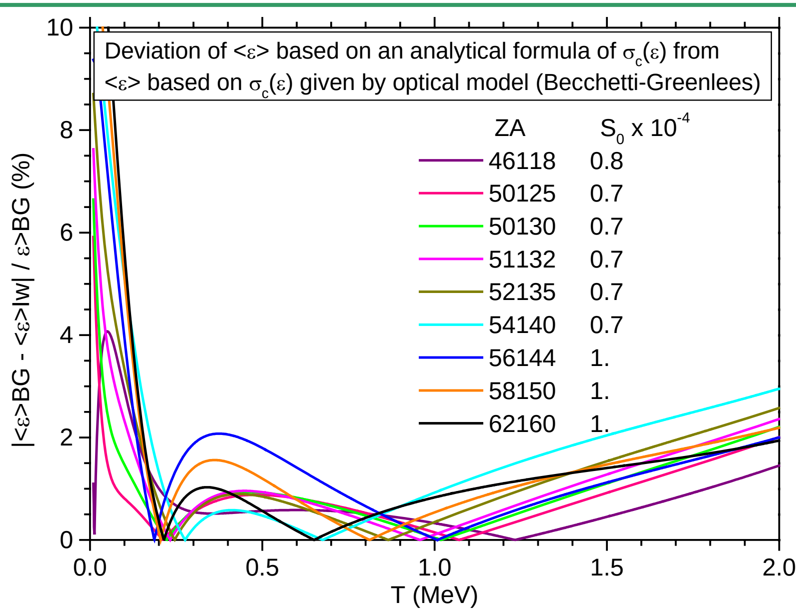
$$\alpha = \frac{\hbar^2}{m\,r_0^2}\frac{S_0}{A^{2/3}}$$

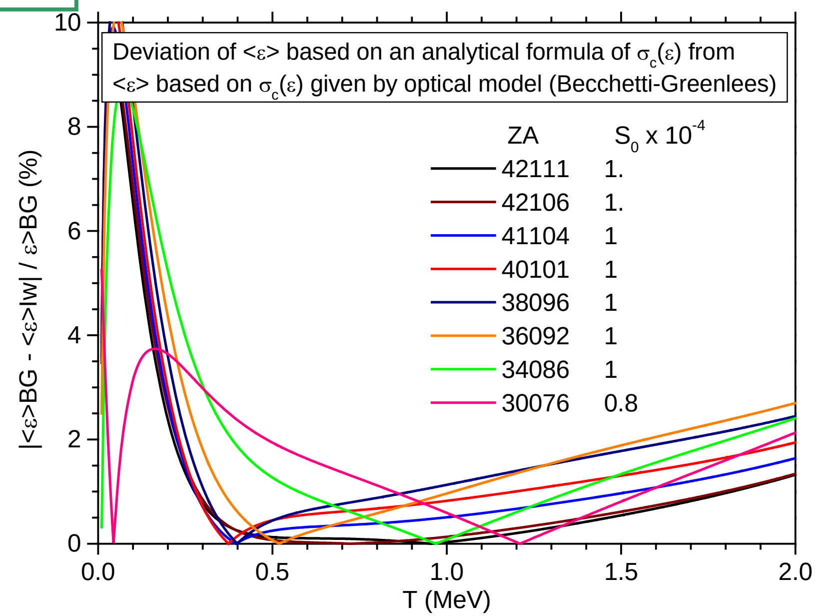
$$K(T) = \left(\sigma_0\,T^{3/2}(\sqrt{T} + \alpha\sqrt{\pi}/2)\right)^{-1}$$

$$\langle\varepsilon\rangle(T) = \frac{T\left(2\sqrt{T} + (3\sqrt{\pi}/4)\alpha\right)}{\sqrt{T} + \alpha\sqrt{\pi}/2}$$

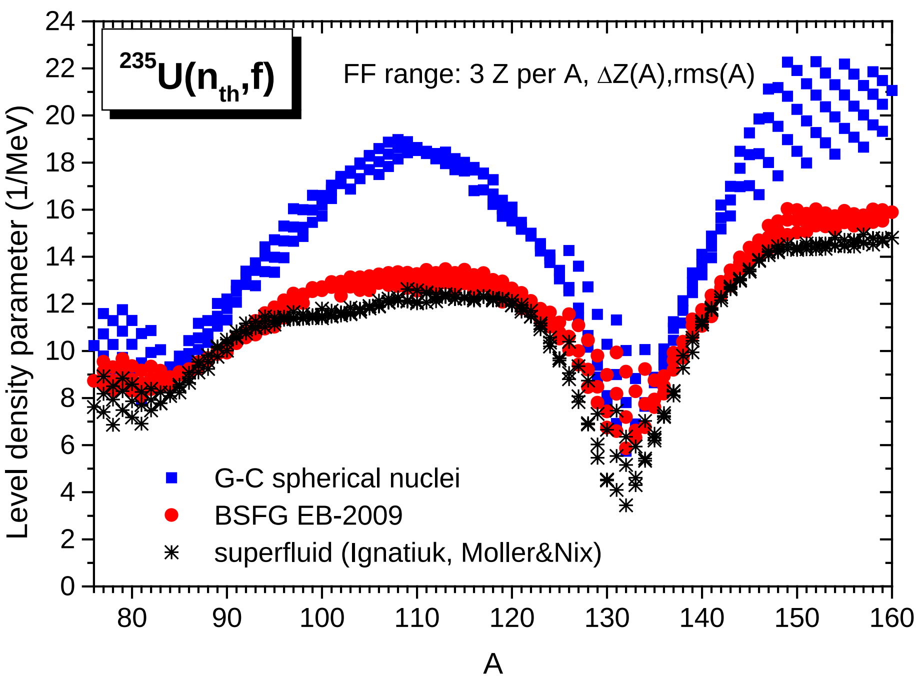
$$deviation(T) = \left|\langle\varepsilon\rangle(T)_{optBG} - \langle\varepsilon\rangle(T)_{analyt.}\right| / \langle\varepsilon\rangle(T)_{optBG}$$

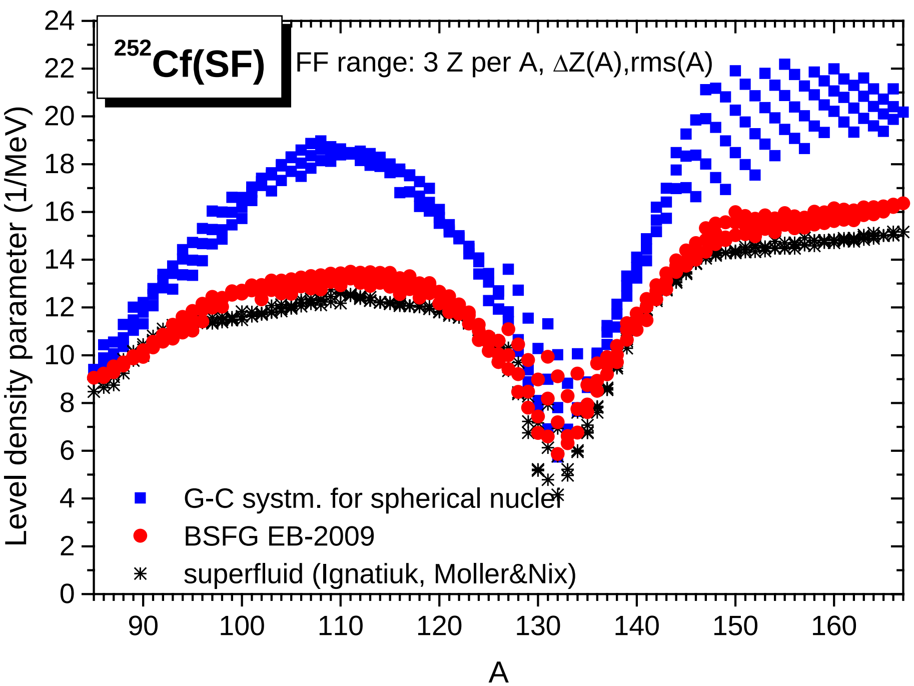
< 4% for T between 0.2 and 2 MeV





Comparison of non-energy dependent level density parameters with the energy-dependent level density parameters of the super-fluid model





Studying the variation with energy of the super-fluid level density parameter of many nuclei appearing as FF, in an energy range going up to about 30 MeV (typical for the residual energies) → the level density parameters given by the E-B systematic for BSFG can approximate the super-fluid level density parameter for a great part of fragments, except the fragments with A around 130, having large negative values of shell corrections (magic or double magic nuclei N=82, Z=50).

PRELIMINARY RESULTS

Detailed calculations following the emission of each prompt neutron done for 3 fissioning nuclei:

$^{235}U(n_{th},f)$
$^{239}Pu(n_{th},f)$
$^{252}Cf(SF)$

Fragmentation range (constructed as in the PbP treatment):

- **A range:** 76 – 160 ($^{235}U(n_{th},f)$), 80 – 160 ($^{239}Pu(n_{th},f)$), 85 – 167 ($^{252}Cf(SF)$), **step 1**
- **3 Z per A as the nearest integers above and below Zp(A) = Z_{UCD}(A) + ΔZ(A)**
- **TKE =** 100 –195 MeV ($^{235}U(n_{th},f)$), 130 – 210 MeV ($^{239}Pu(n_{th},f)$), 140 – 210 MeV ($^{252}Cf(SF)$), **with a step of 5 MeV**

$$Y(A,Z,TKE) = p(Z,A)Y_{\exp}(A,TKE)$$

- **p(Z,A): Gaussian centered on Zp(A) with rms(A)** (ΔZ(A), rms(A) ZP model)
- **Experimental Y(A,TKE) measured at JRC-Geel:**
  - $^{235}U(n_{th},f)$: Al-Adili et al.
  - $^{239}Pu(n_{th},f)$: Wagemans et al.
  - $^{252}Cf(SF)$: Göök et al.

For each fragmentation at each TKE – probability Y(A,Z,TKE) with the initial complementary fragments: (A, Z, TKE) and (A_0-A, Z_0-Z, TKE)

- **number of emitted neutrons: k_{max}(A,Z,TKE)**
- **initial fragment (Z,A) and residual nuclei (Z,A-k+1) with k = 1 to k_{max}(A,Z,TKE)**

- **TXE(A,Z,TKE), E*(A,Z,TKE) (from TXE partition based on modeling at scission)**
- **Sn (Z,A-k+1), a(Z,A-k+1) (non-energy dependent, BSFG systm. EB-2009)**

$$\sigma_c(Z, A-k+1, \varepsilon) = \sigma_0^{(k)}\left(1+\alpha_k/\sqrt{\varepsilon}\right)$$ **with $\sigma_0^{(k)}$ and α_k depending on Z, A-k+1**

- **$T_r^{(k)}$(A,Z,TKE), $E_r^{(k)}$(A,Z,TKE), $<\varepsilon_k>(T_r^{(k)}$,A,Z,TKE) etc.**
- **distributions $P(T_r^{(k)})$, $P(E_r^{(k)})$ etc. following the emission of each neutron k**
- **the sum of these distrib. following the successive emission of all neutr. (HF, LF, all)**

$$q(A,Z,TKE) = \frac{1}{k_{max}(A,Z,TKE)} \sum_{k=1}^{k\max(A,Z,TKE)} q_k(A,Z,TKE)$$

Quantity as a function of initial fragment and TKE:

Example: $$\nu(A,Z,TKE) = \frac{1}{k_{max}(A,Z,TKE)} \sum_{k=1}^{k\max(A,Z,TKE)} k$$

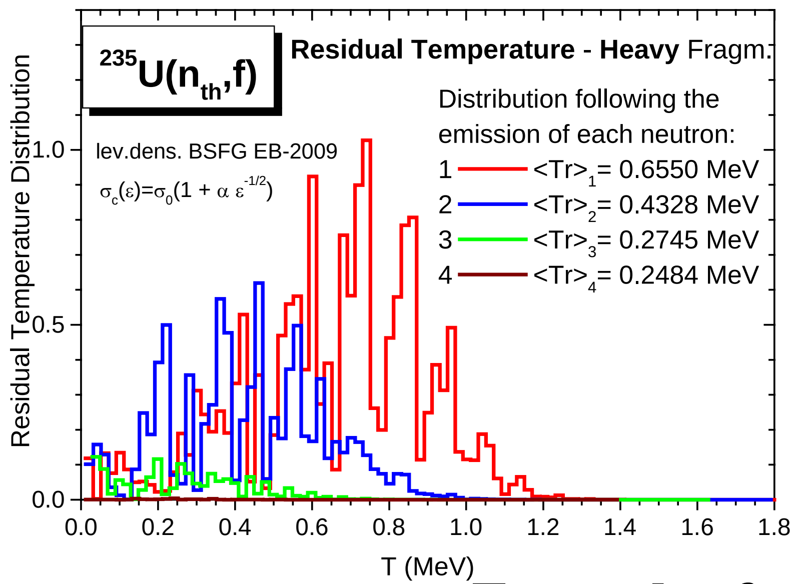
$$\bar{q}_k(A) = \sum_{Z,TKE} q_k(A,Z,TKE)Y(A,Z,TKE) \Big/ \sum_{Z,TKE} Y(A,Z,TKE)$$

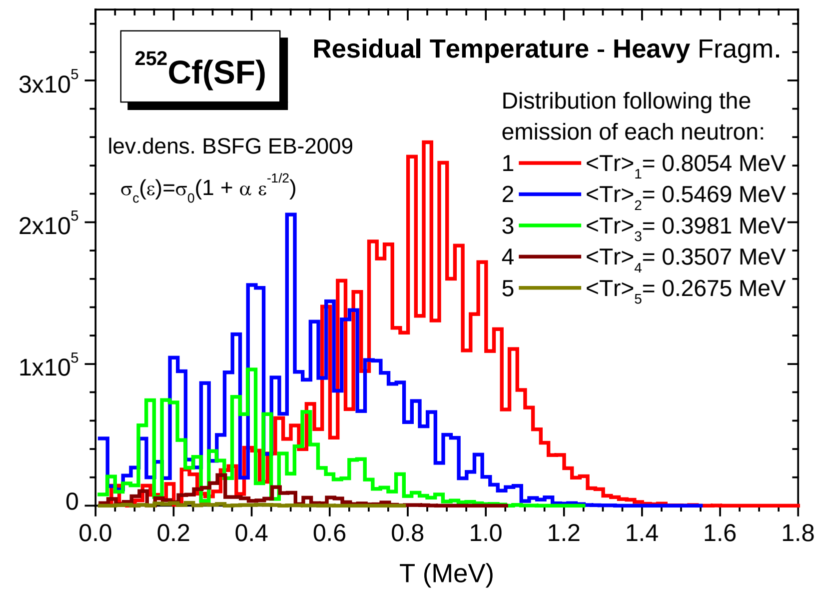
Quantity corresponding to the emission of each neutron as a func. of A, or of TKE etc.

$$\bar{q}(A) = \sum_{Z,TKE}\left(\frac{1}{k_{max}(A,Z,TKE)} \sum_{k=1}^{k_{max}(A,Z,TKE)} q_k(A,Z,TKE)\right)Y(A,Z,TKE) \Big/ \sum_{Z,TKE} Y(A,Z,TKE)$$

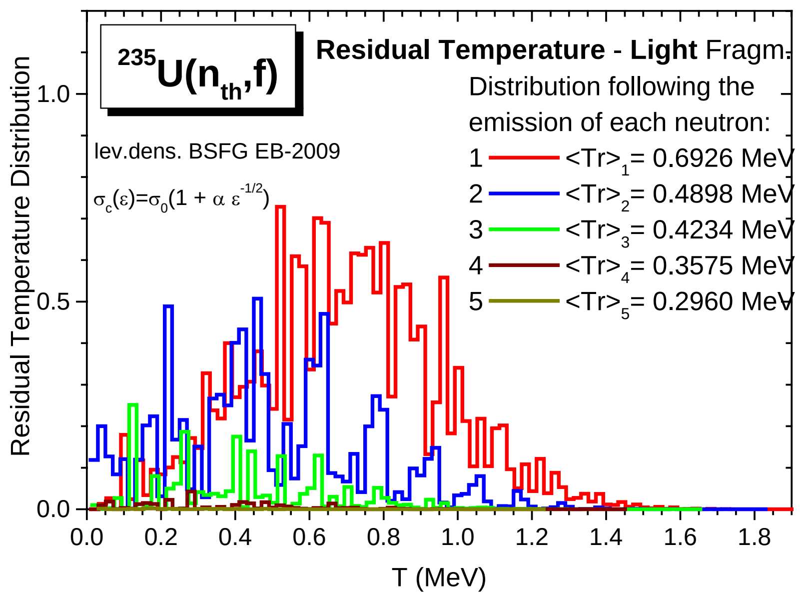
Residual temperature distribution following the emission of each neutron

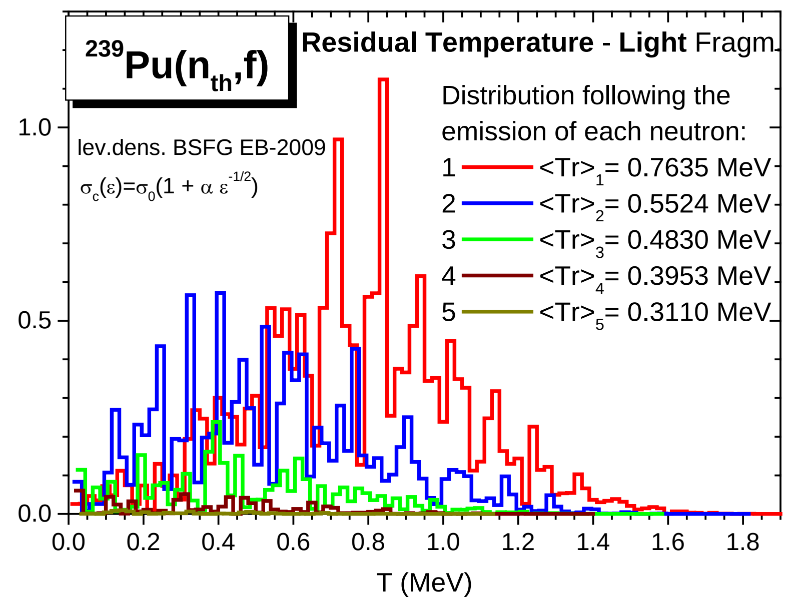
Examples for Heavy Fragments





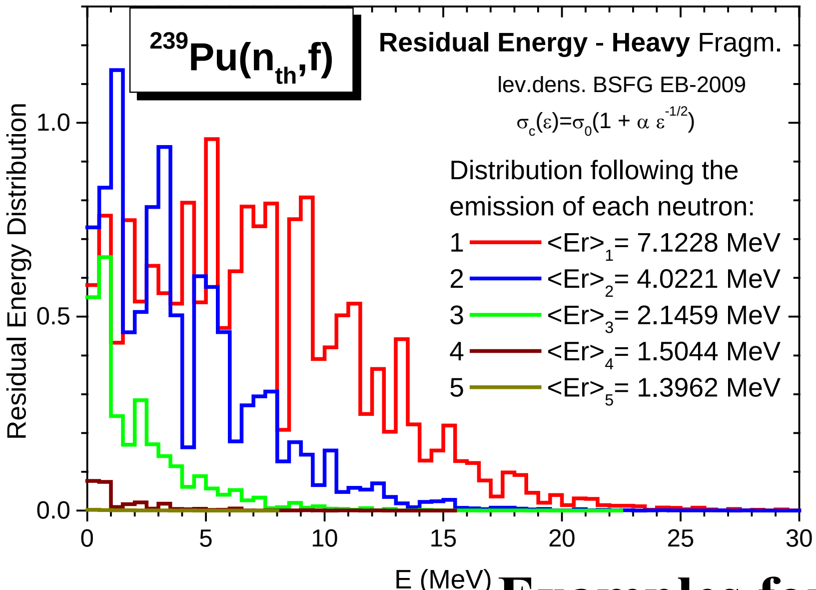
Examples for Light Fragments

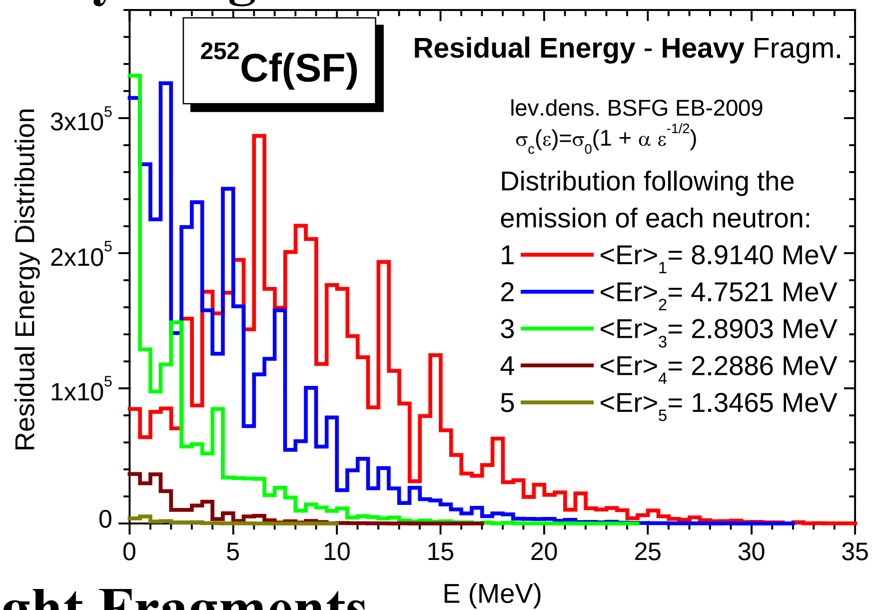




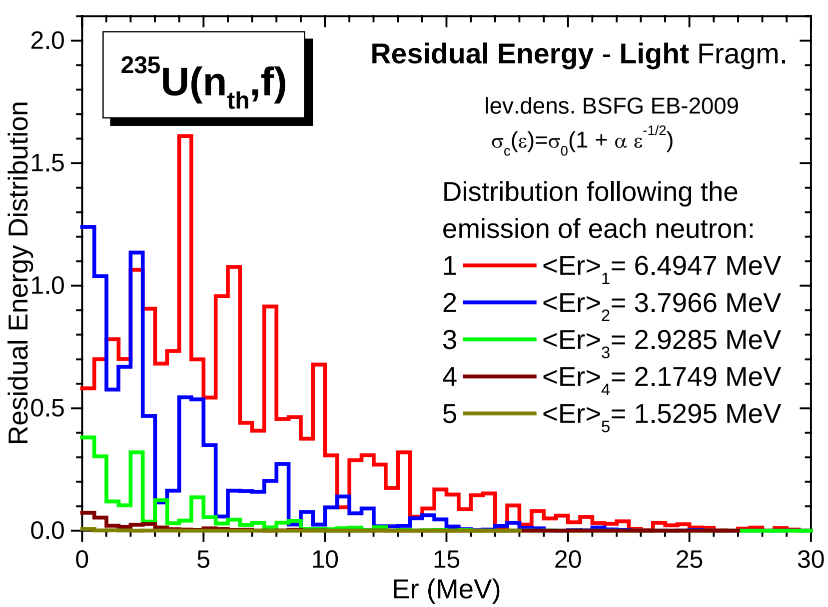
Residual energy distribution following the emission of each neutron

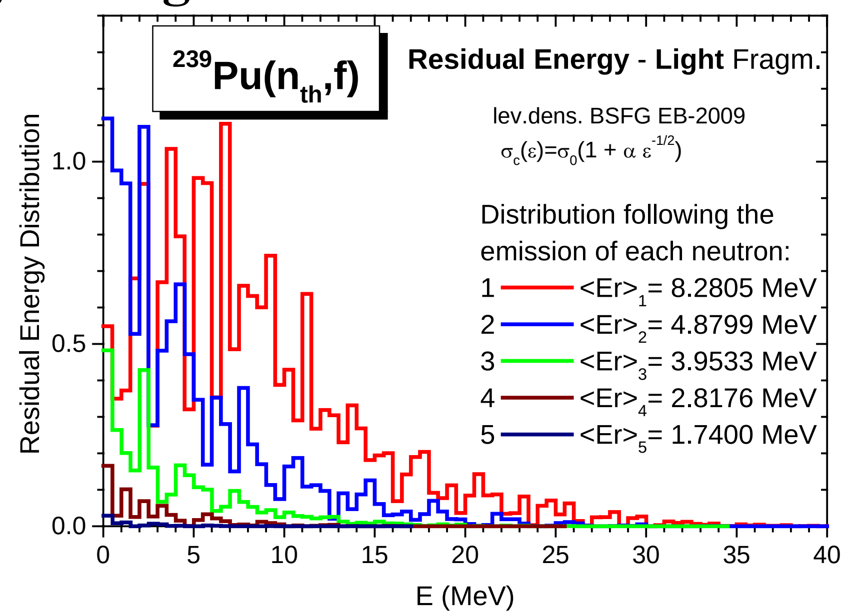
Examples for Heavy Fragments





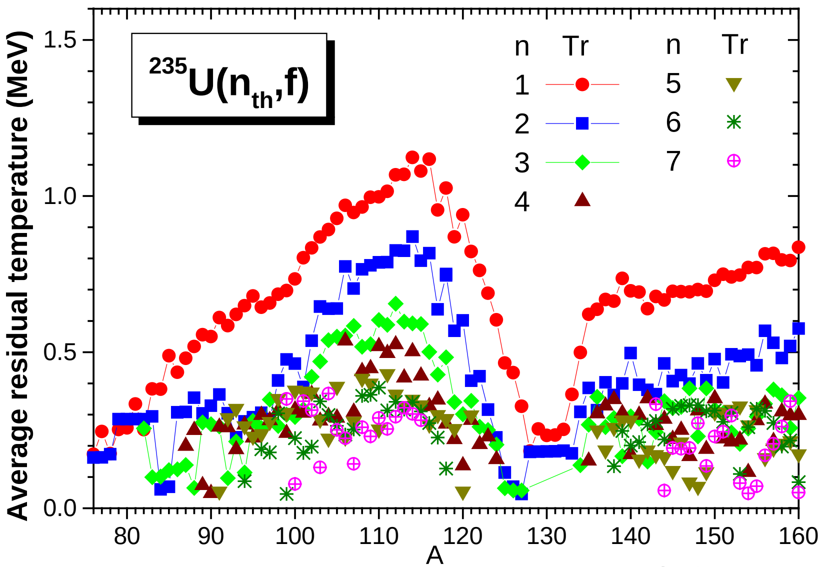
Examples for Light Fragments

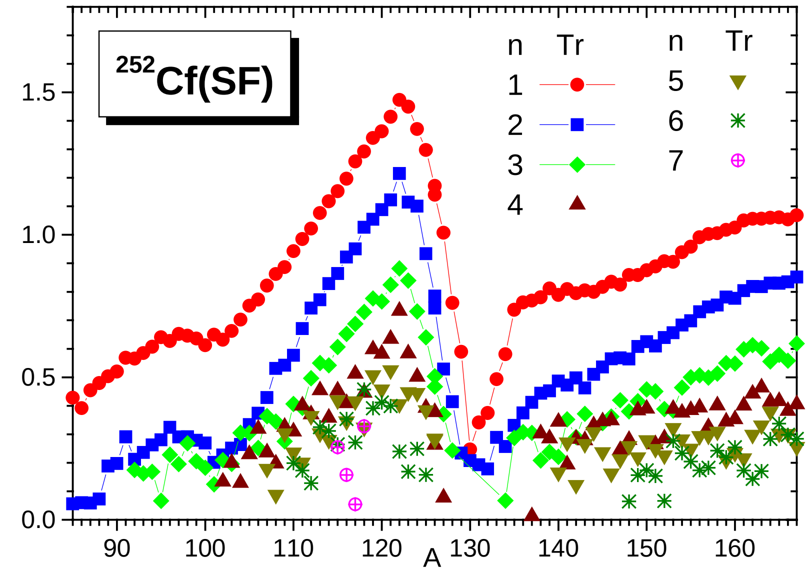




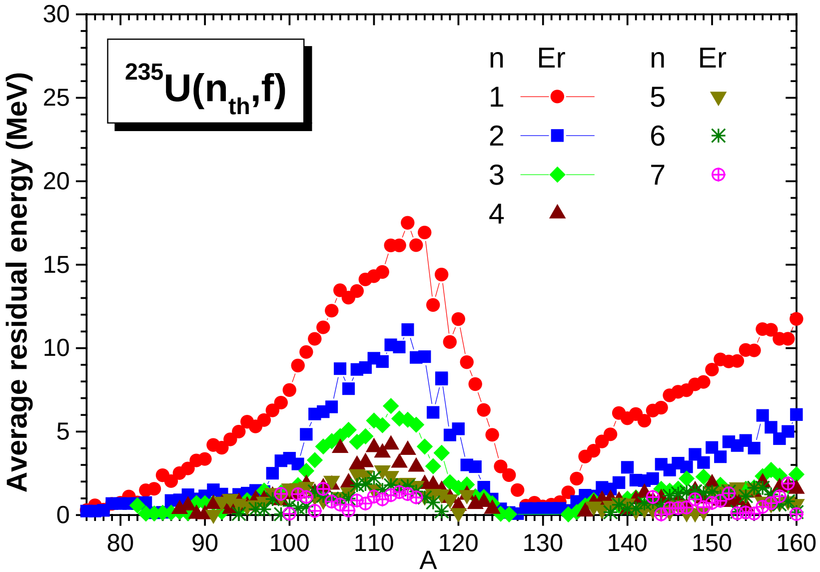
Average quantities following the emission of each neutron as a function of initial fragment mass

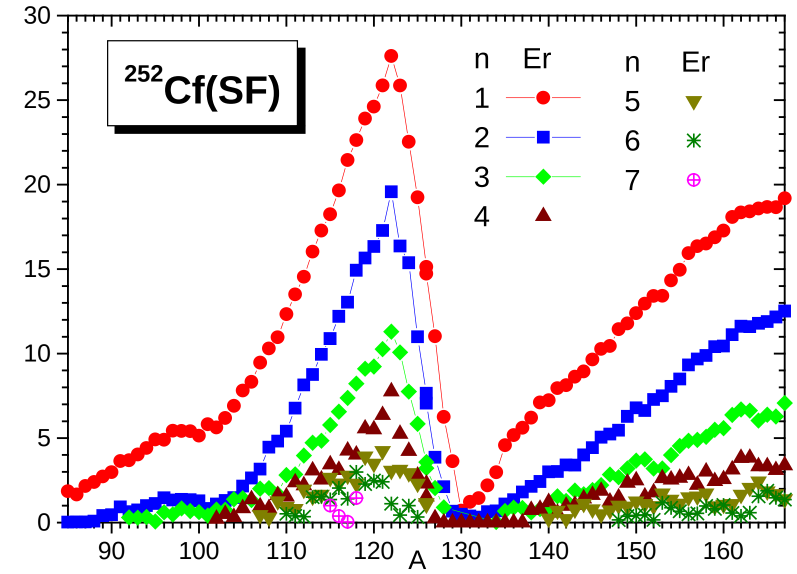
Examples of Average Residual Temperature



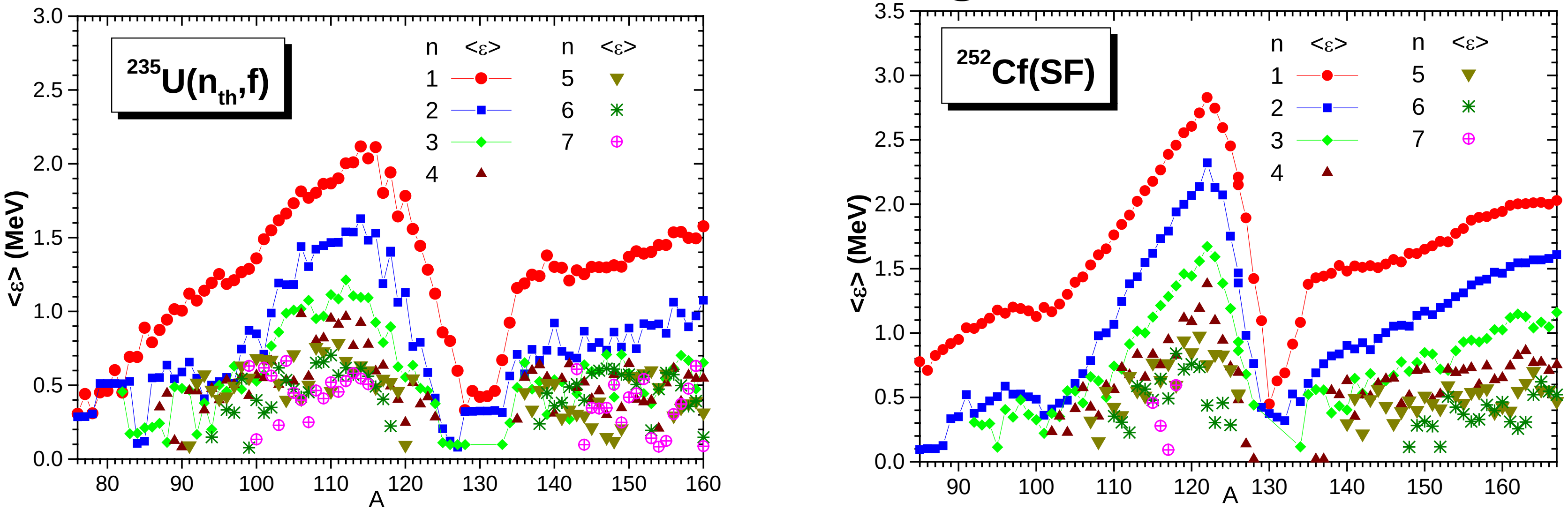


Examples of Average Residual Energy

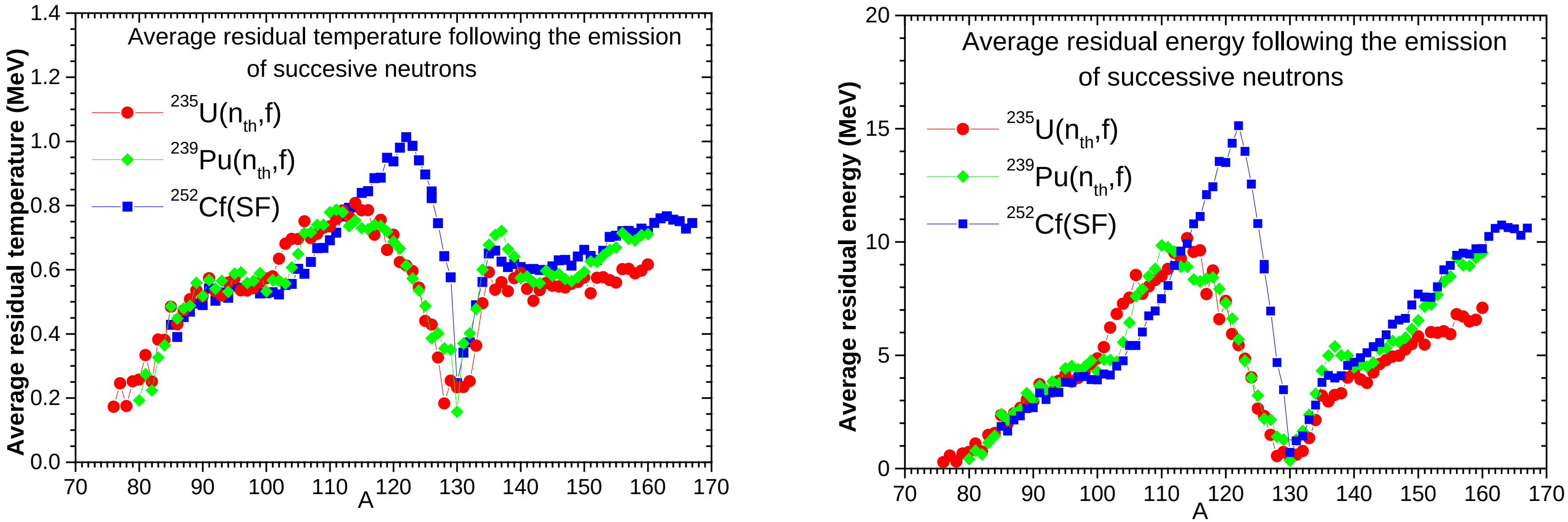




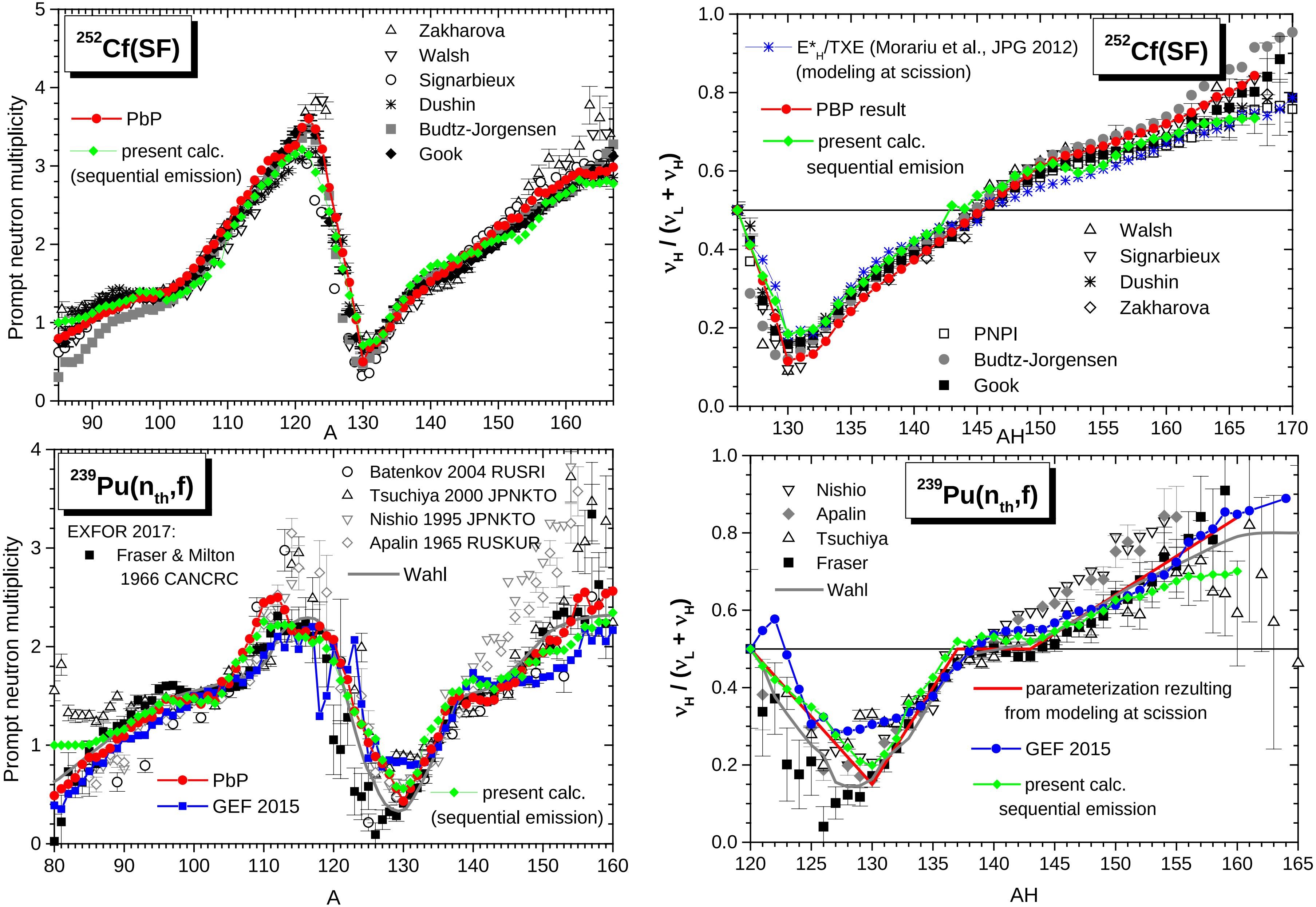
Average energy in the center-of-mass frame of each emitted prompt neutron as a function of initial fragment mass



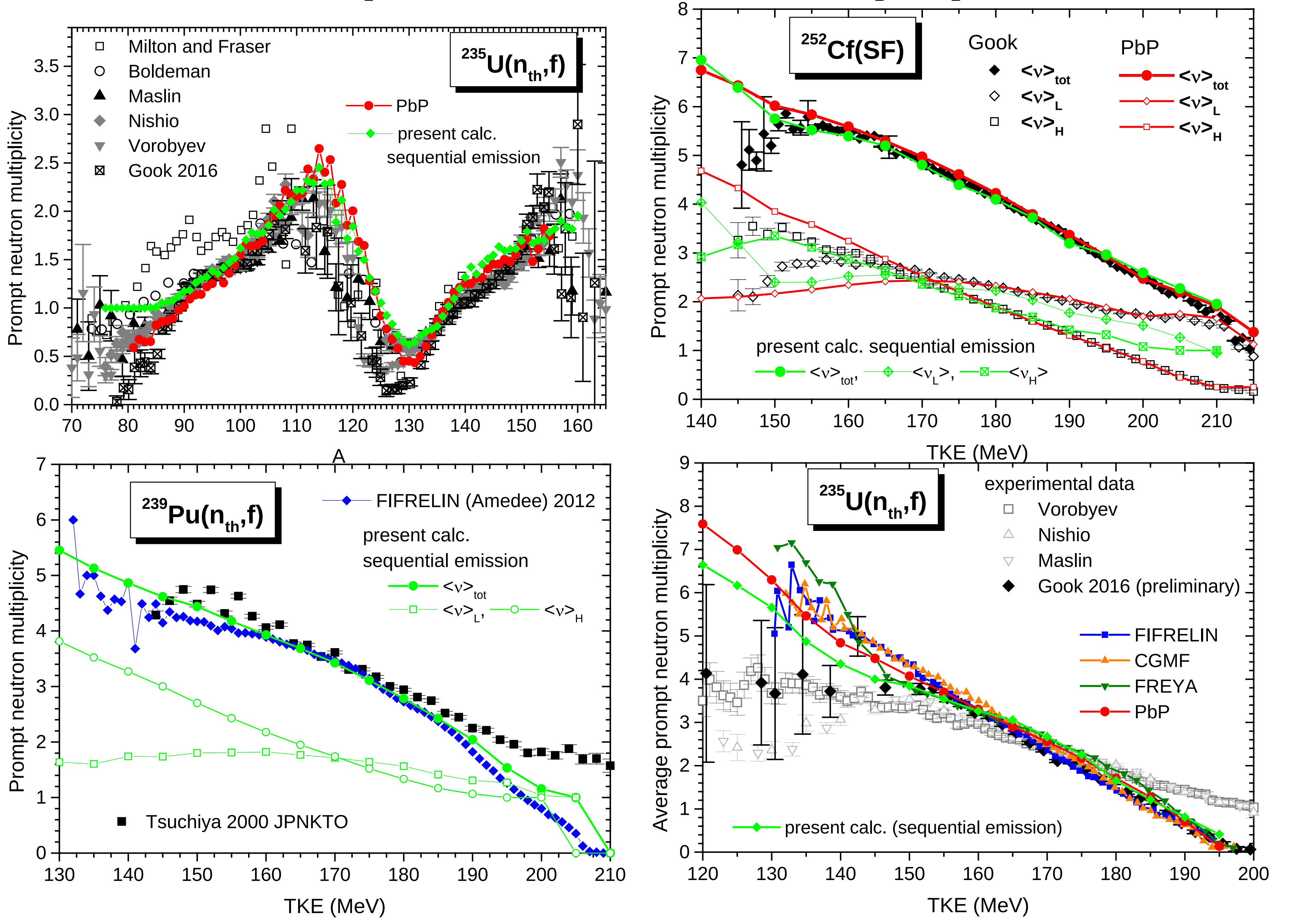
Average Trez and Erez following the successive emission of all neutrons as a function of the initial fragment mass



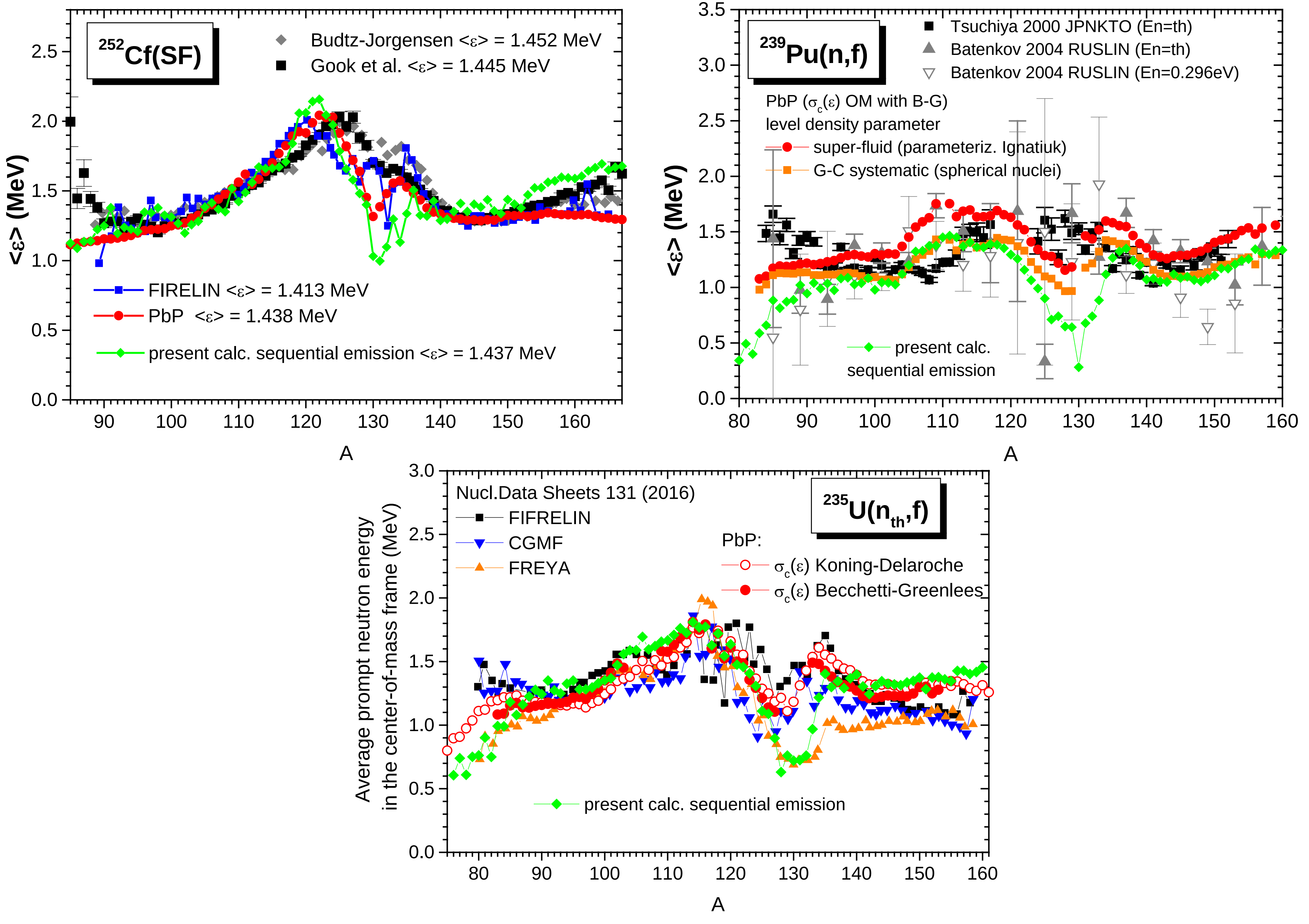
Verification of present results (sequential emission) with experimental data and results of other prompt emission models



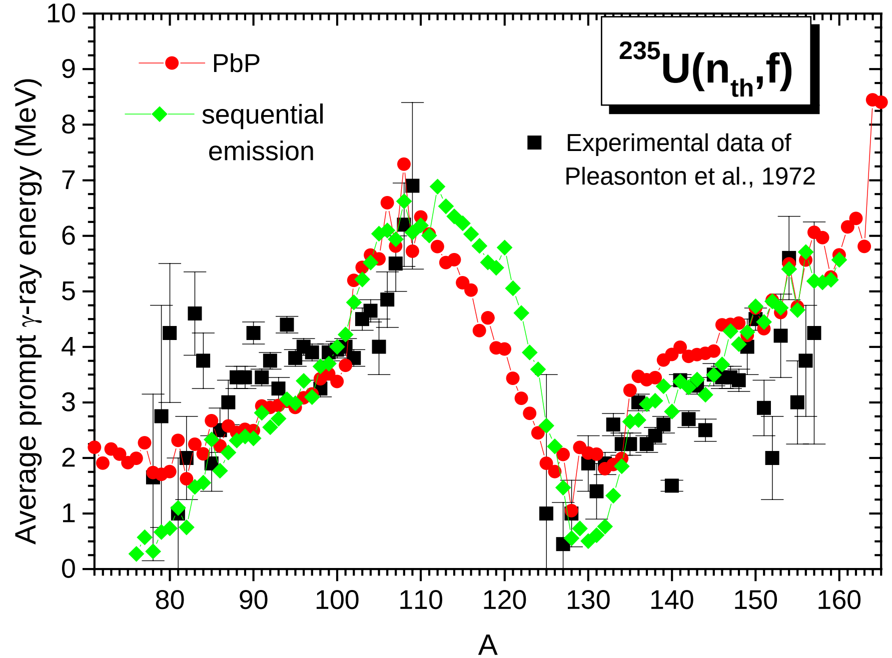
Verification with experimental data and results of other prompt emission models

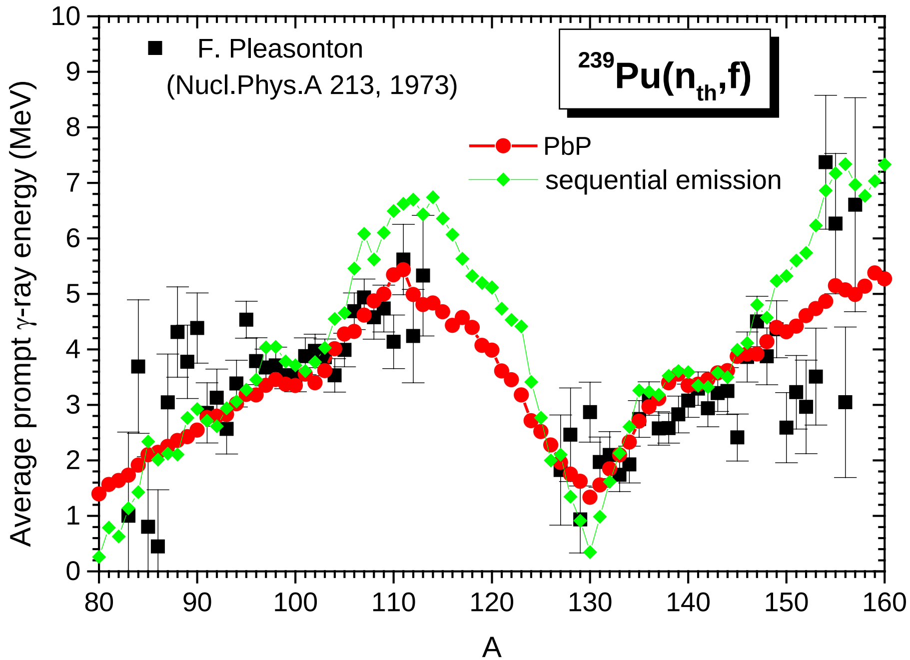


Verification with experimental data and results of other prompt emission models

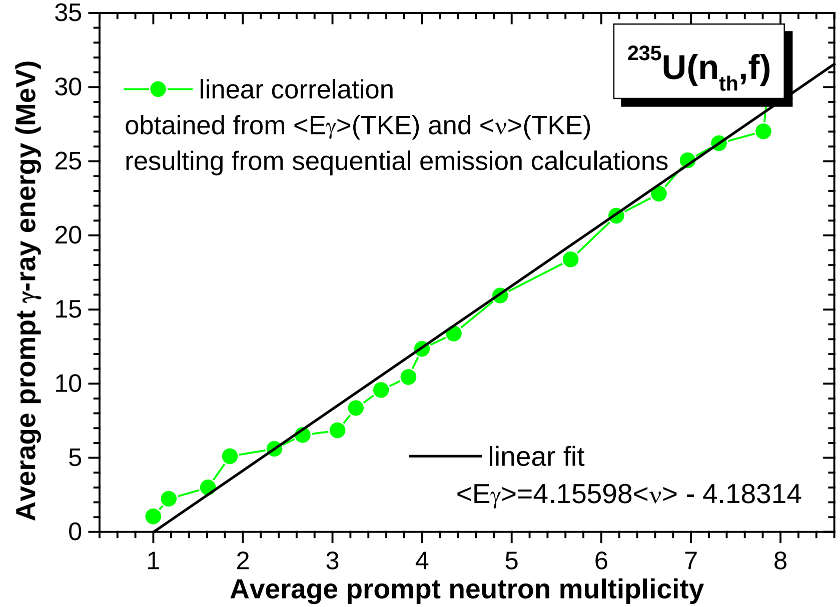


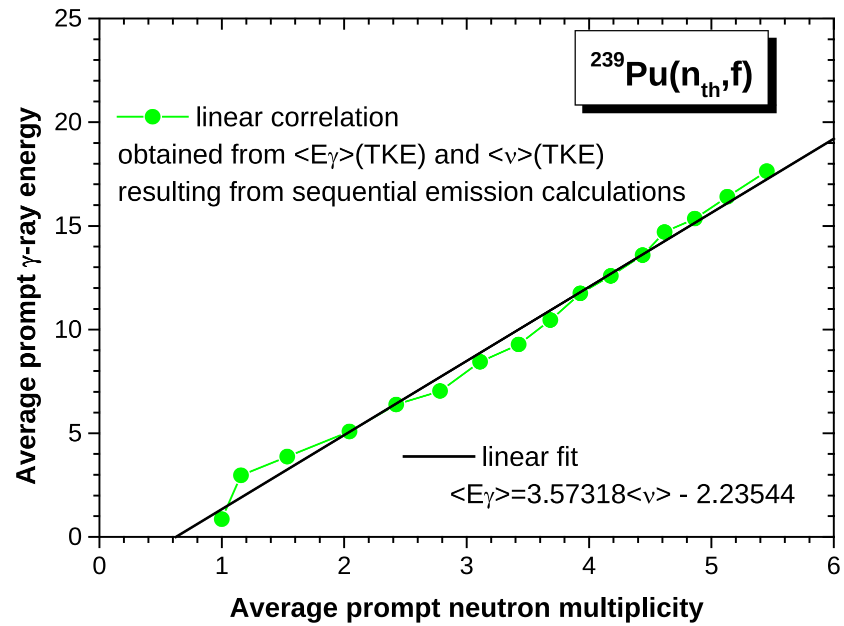
<Eγ>(A) comparison with exp. data



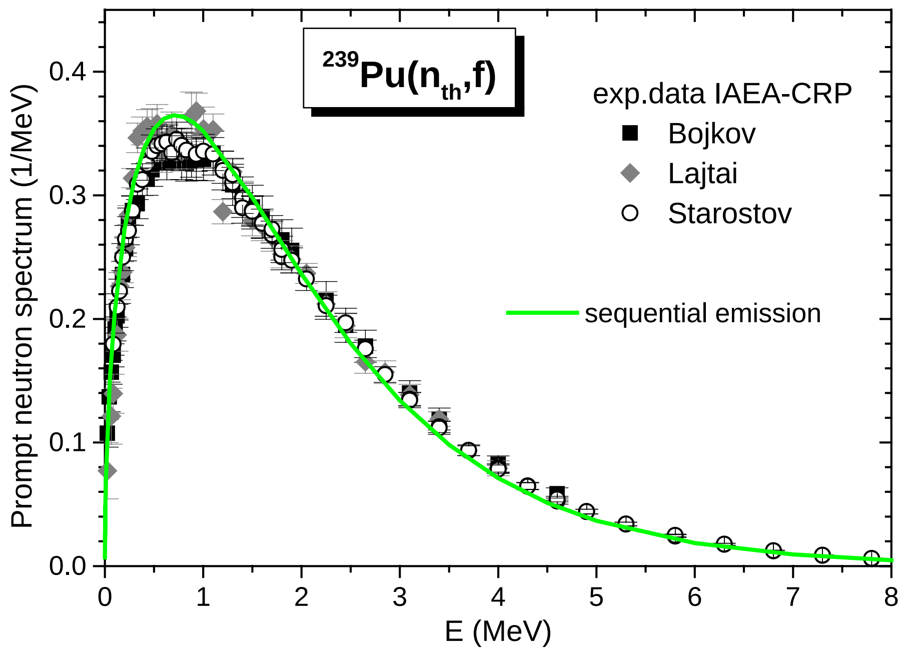


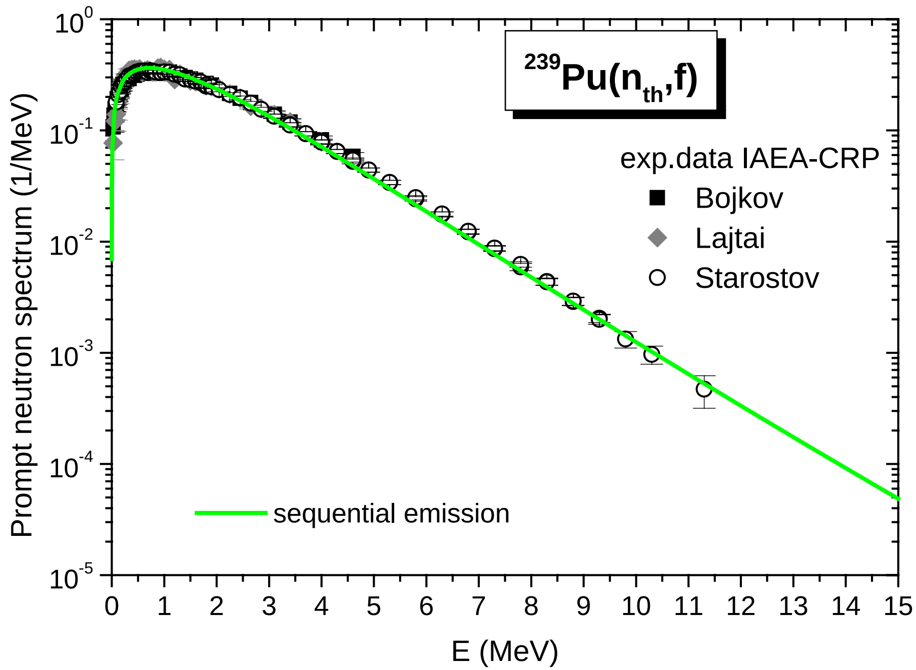
Linear correlation between <Eγ> and <ν>





Prompt neutron spectra in the laboratory frame - preliminary results





for each A,Z,TKE, the spectrum in CMS of the emitted k-th neutron (with k = 1 to kmax):

$$\varphi_k(\varepsilon) = \frac{\left(\varepsilon + \alpha_k\sqrt{\varepsilon}\right)\exp(-\varepsilon / T_k)}{T_k^{3/2}\left(\sqrt{T_k} + \alpha_k\sqrt{\pi}/2\right)}$$

The average spectrum corresponding to (A,Z,TKE):

$$\overline{\varphi}(\varepsilon, A, Z, TKE) = \frac{1}{k_{\max}(A,Z,TKE)}\sum_{k=1}^{k\max}\varphi_k(A,Z,TKE)$$

In the Laboratory frame:

$$N(E,A,Z,TKE) = \frac{1}{4\sqrt{E_f(A,Z,TKE)}}\int_{u1}^{u2}\overline{\varphi}(\varepsilon,A,Z,TKE)\frac{d\varepsilon}{\sqrt{\varepsilon}}$$

$$u_{1,2}(A,Z,TKE) = \left(\sqrt{E} \mp \sqrt{E_f(A,Z,TKE)}\right)^2$$

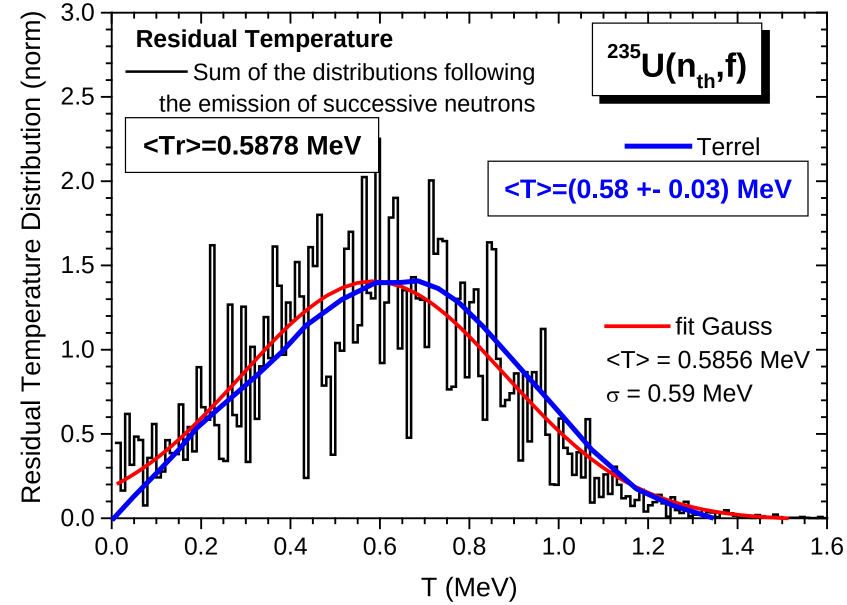
$$E_f(A,Z,TKE) = \frac{A_0 - A}{A}\frac{TKE}{A_0}$$

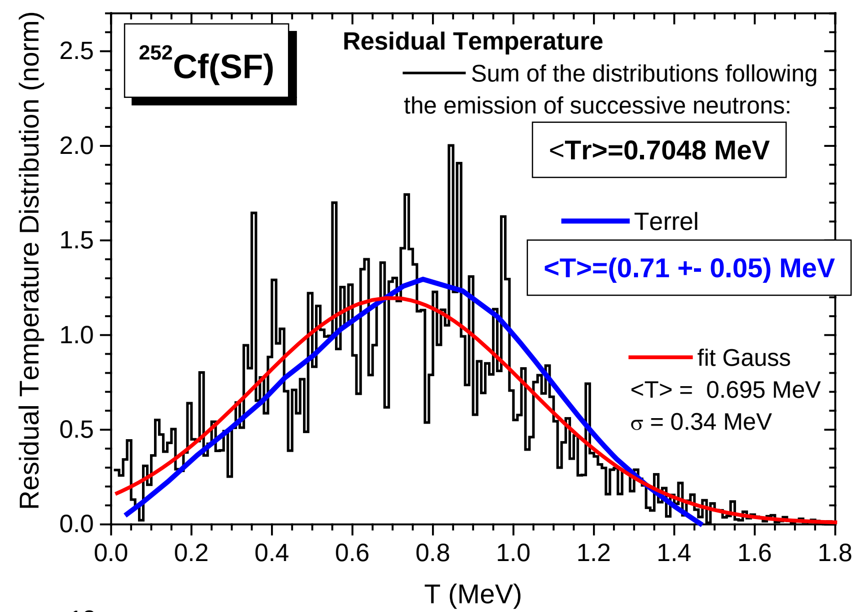
For each fragmentation (pair of FF) at each TKE:

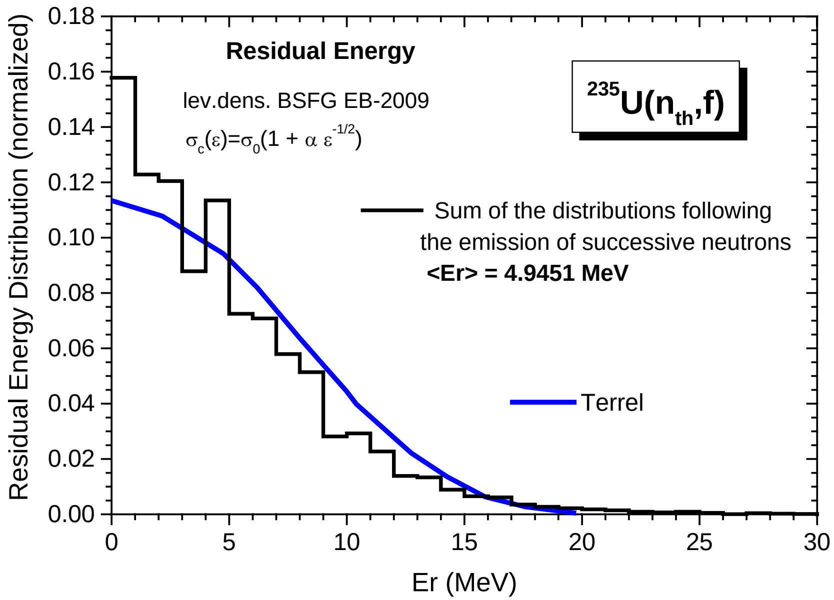
$$N_{pair}(E) = \frac{\nu_L}{\nu_L + \nu_H}N_L(E) + \frac{\nu_H}{\nu_L + \nu_H}N_H(E)$$

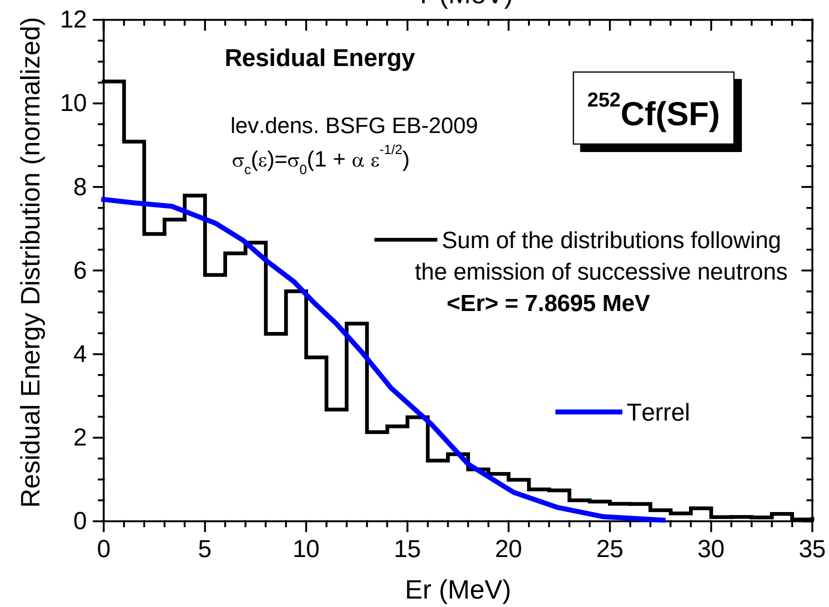
This Npair(E) is averaged over Y(A,Z,TKE) giving the total PFNS in the lab. frame

Sum of the Trez and Erez distributions following the emission of successive neutrons from all fragments – comparison with the results of Terrel





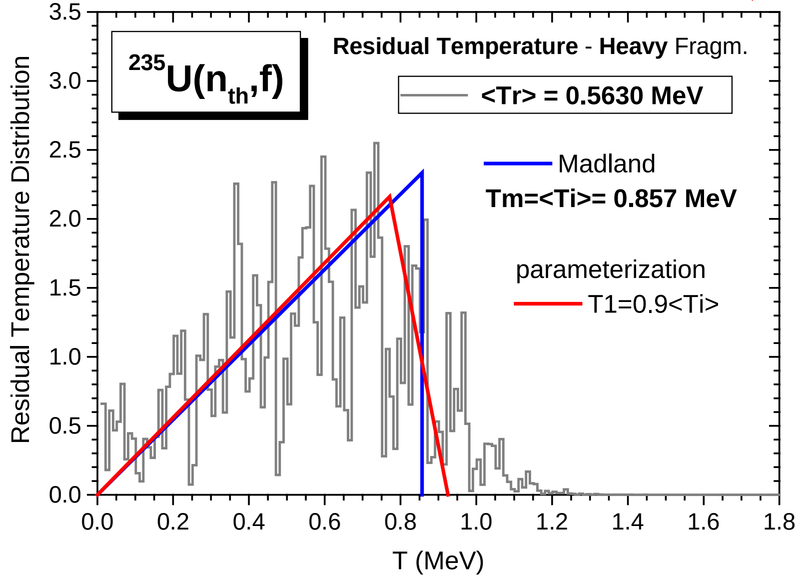


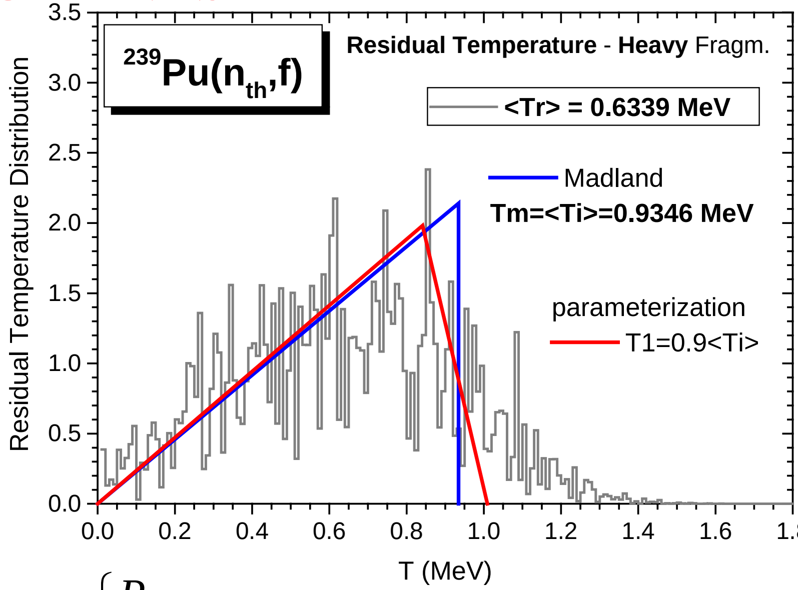


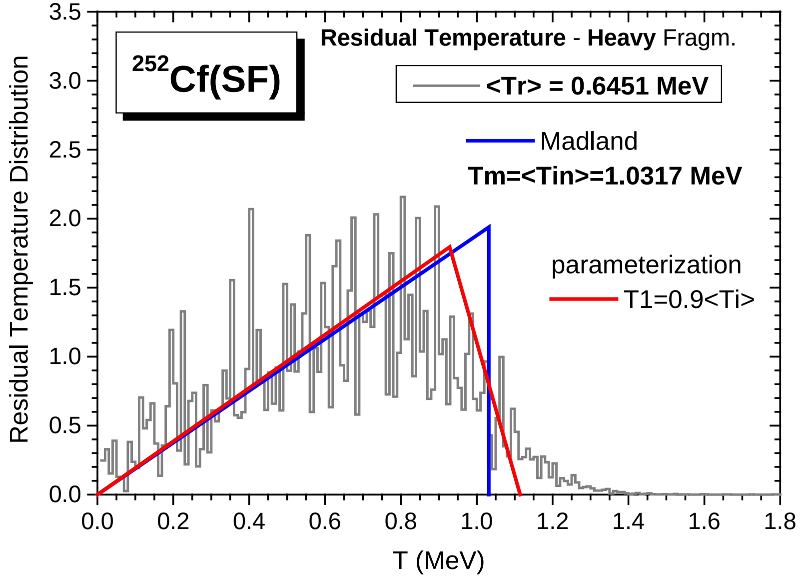
Preliminary new form of the residual temperature distribution P(T)

Parameterization as a function of the average temperature of initial fragments <Ti>

HEAVY FRAGMENTS







$$P(T) = \begin{cases} \dfrac{P_{\max}}{T_1} T & T \le T_1 \\ \dfrac{P_{\max}}{(T_2 - T_1)}(T_2 - T) & T_1 \le T \le T_2 \end{cases}$$

$$\int_0^{T2} P(T)dT = 1 \rightarrow P_{\max} = 2/T_2$$

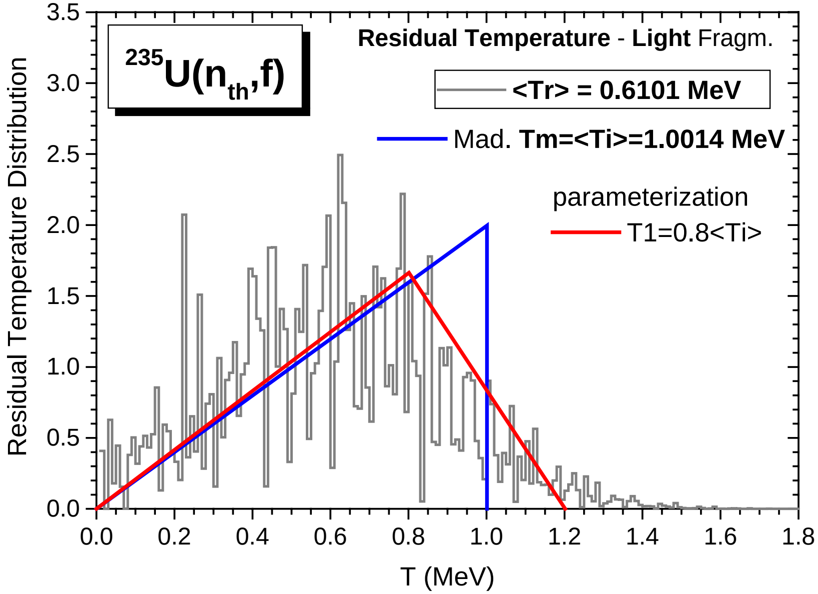
$$< Tr >= \int_0^{T2} T\,P(T)dT = (T_1 + T_2)/3 \rightarrow T_2 = 3 < Tr > - T_1$$

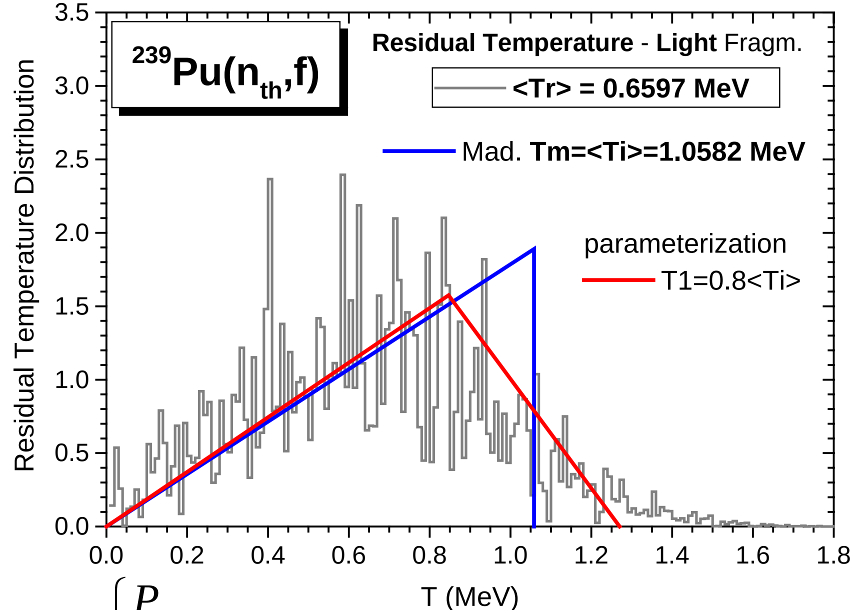
$$T_1 = 0.9 < T_i > \quad < Tr > \approx 0.66 < T_i >$$

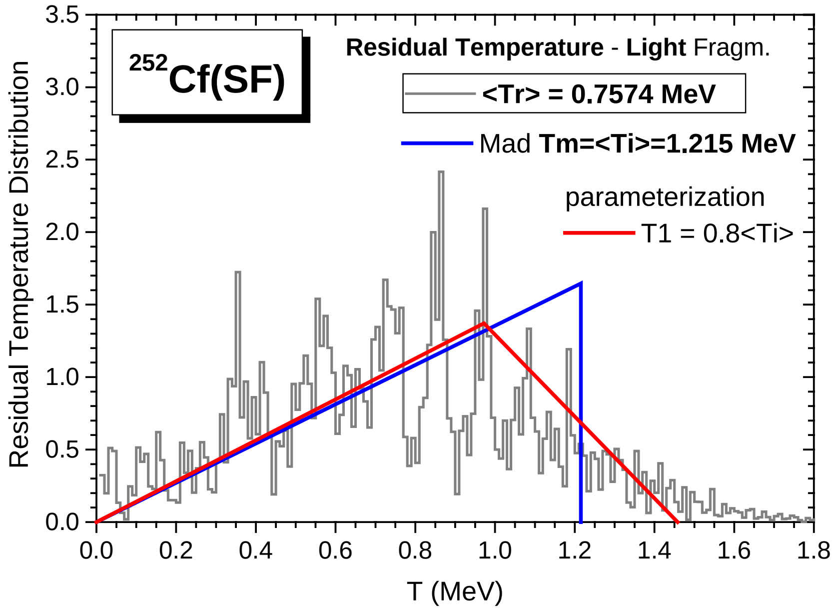
Preliminary new form of the residual temperature distribution P(T)

Parameterization as a function of the average temperature of initial fragments <Ti>

LIGHT FRAGMENTS







$$P(T)=\begin{cases}\dfrac{P_{\max}}{T_1}T & T\le T_1\\[2ex] \dfrac{P_{\max}}{(T_2-T_1)}(T_2-T) & T_1\le T\le T_2\end{cases}$$

$$\int_0^{T2}P(T)dT=1\;\rightarrow\quad P_{\max}=2/T_2$$

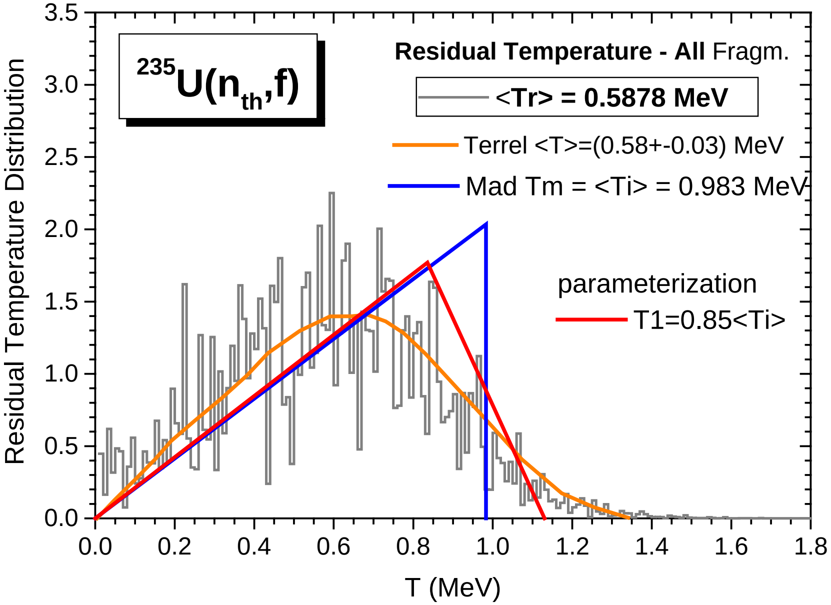
$$<Tr>=\int_0^{T2}T\,P(T)dT=(T_1+T_2)/3\;\rightarrow T_2=3<Tr>-T_1$$

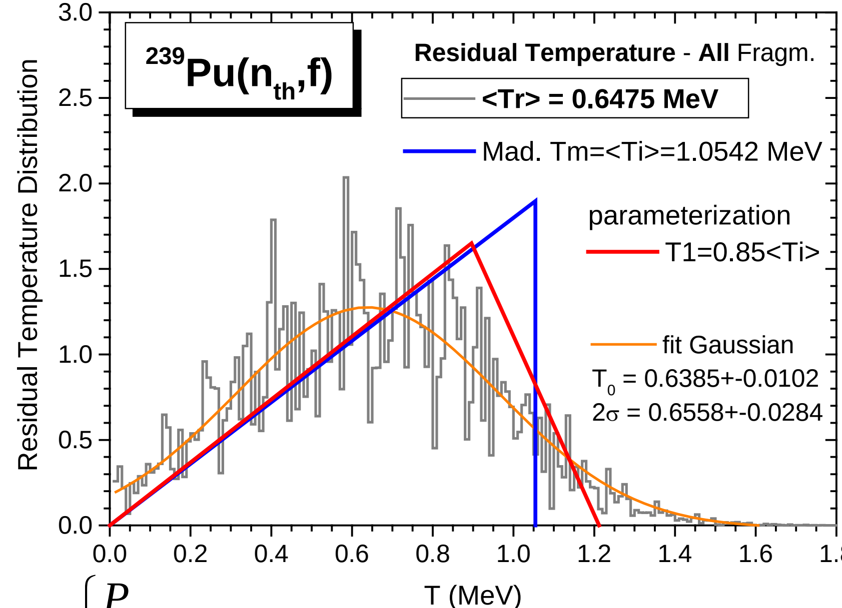
$$T_1=0.8<T_i>\qquad <Tr>\approx 0.66<T_i>$$

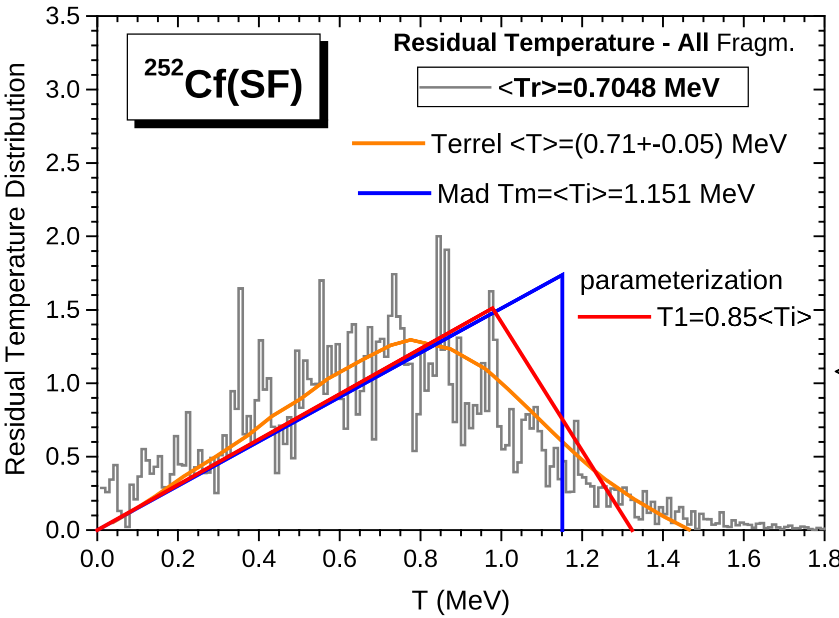
Preliminary new form of the residual temperature distribution P(T)

Parameterization as a function of the average temperature of initial fragments <Ti>

ALL FRAGMENTS







$$P(T) = \begin{cases} \dfrac{P_{\max}}{T_1} T & T \le T_1 \\ \dfrac{P_{\max}}{(T_2 - T_1)}(T_2 - T) & T_1 \le T \le T_2 \end{cases}$$

$$\int_0^{T2} P(T)dT = 1 \;\rightarrow\; P_{\max} = 2/T_2$$

$$< Tr >= \int_0^{T2} T\,P(T)dT = (T_1 + T_2)/3 \;\rightarrow\; T_2 = 3 < Tr > - T_1$$

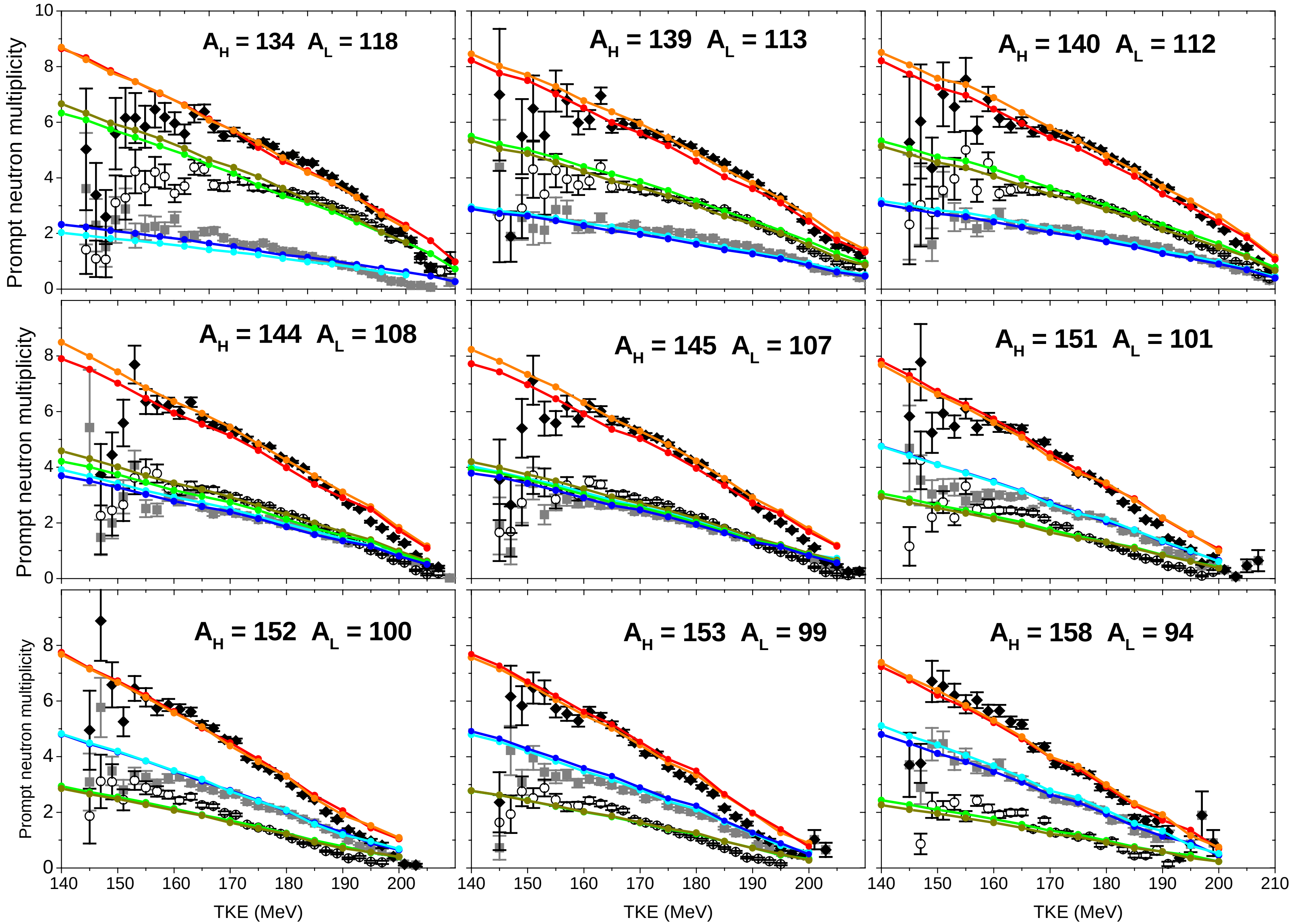
$$\boxed{T_1 = 0.85 < T_i > \quad < Tr > \approx 0.66 < T_i >}$$

Results of the PbP model with the preliminary parameterization of P(T)

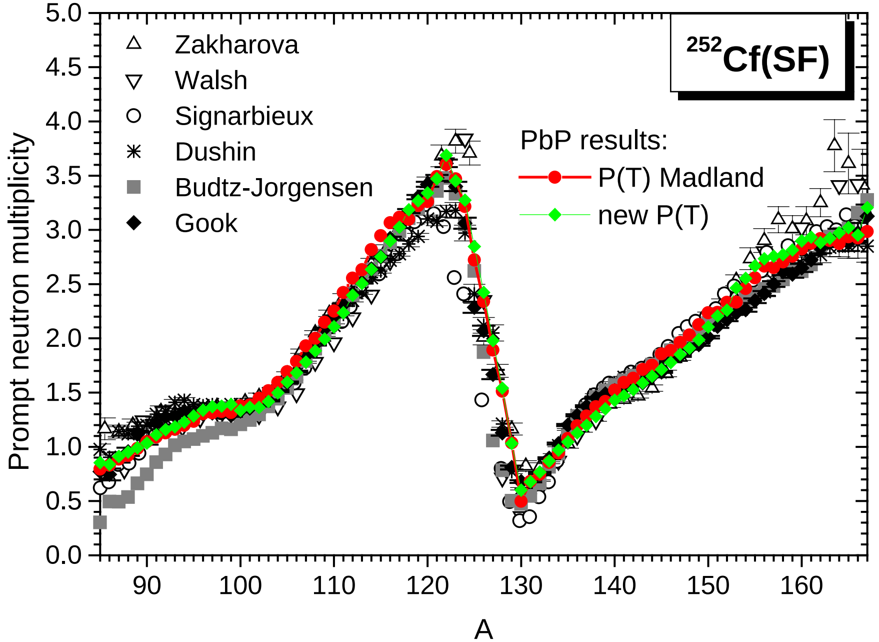
new P(T): orange, dark yellow, cyan, P(T) Madland: red, green, blue

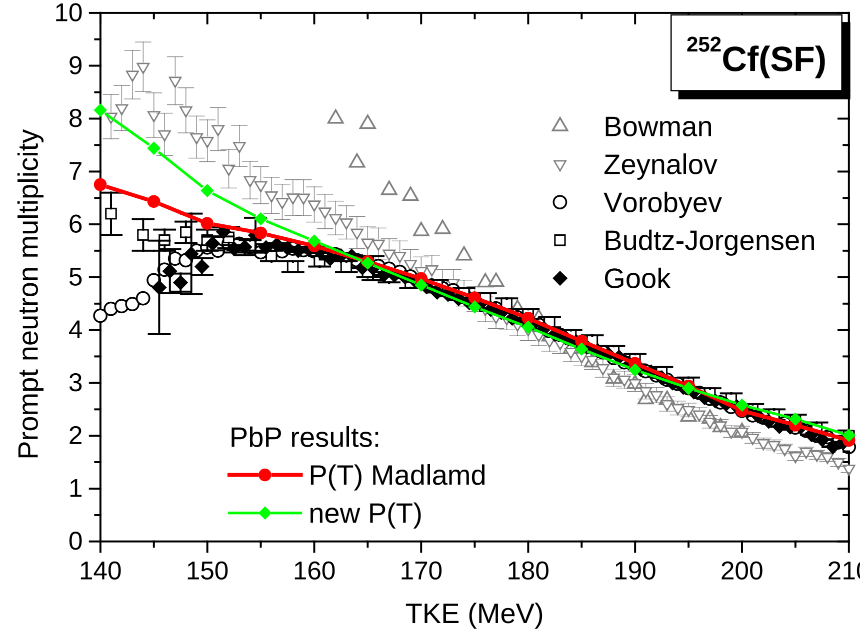
Here only mass pairs for which the differences are visible

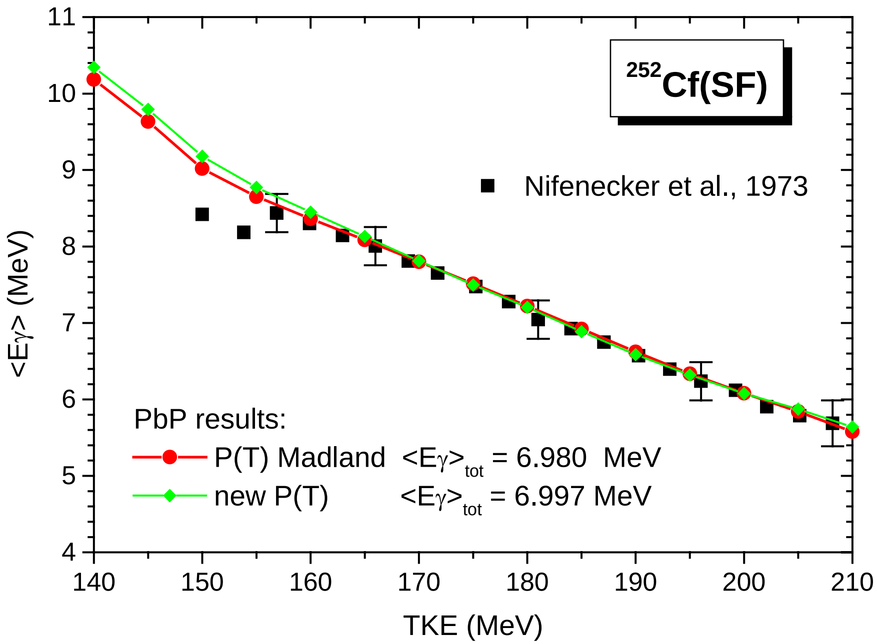
^{252}Cf(SF)

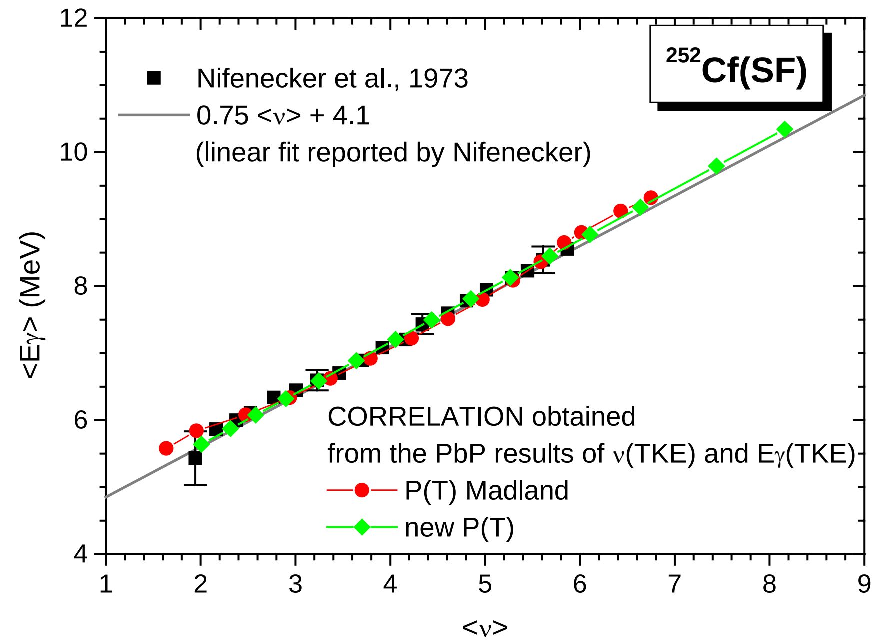


Results of the PbP model with the preliminary parameterization of P(T)

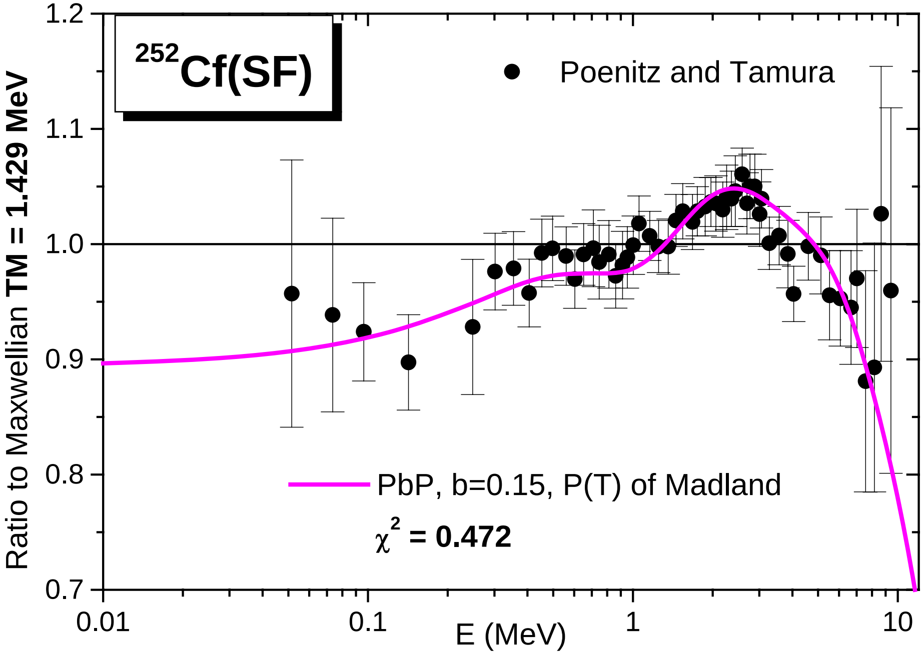








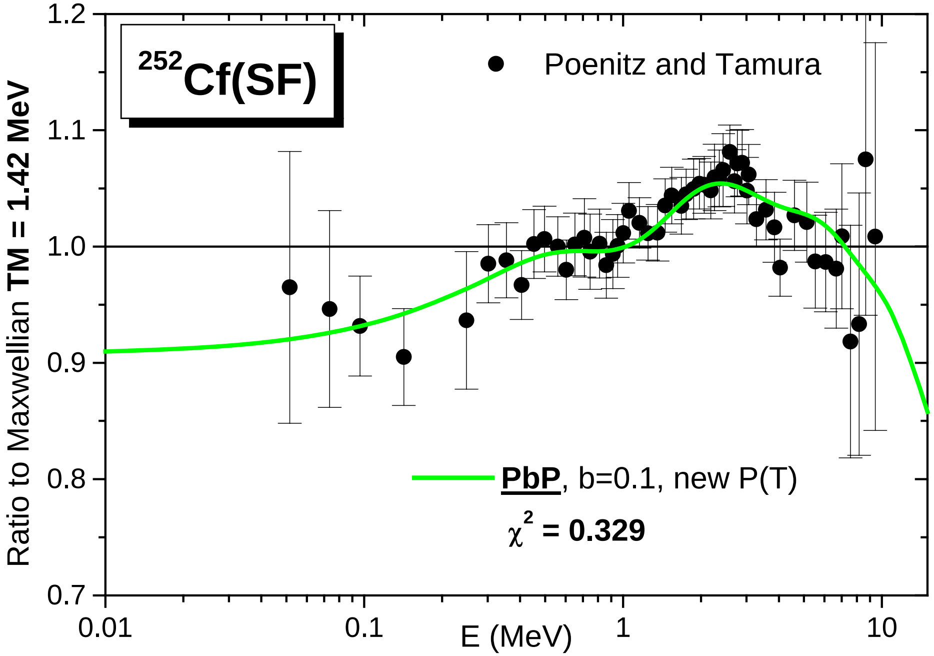
P(T) of Madland and Nix



$$P(T) = \begin{cases} \frac{2}{T_i^{\,2}}\; T & T \le T_i \\ 0 & T > T_i \end{cases}$$

$$< T > = \frac{2}{3} T_i$$

new parameterization of P(T) (preliminary)



$$P(T) = \begin{cases} \frac{2}{T_i^{\,2}} \alpha T & T \le a T_i \\ \frac{2}{T_i^{\,2}} \left(\beta T_i - \gamma T \right) & a T_i \le T \le (2-a) T_i \end{cases}$$

$$1/\alpha = a(2-a)$$

$$1/\beta = 2(1-a)$$

$$1/\gamma = 2(1-a)(2-a)$$

$$< T > \approx 0.66 T_i = \frac{2}{3} T_i$$

CONCLUSIONS

- **The very good description of the experimental ν(A,TKE) matrix of Göök et al. by the PbP results (with both P(T) the "classical" triangular form of Madland and Nix and the new preliminary parameterization) validates the PbP model itself (i.e. without the implication of Y(A,TKE) distributions).**

- **The detailed calculations taking into account the successive emission of prompt neutrons (sequential emission) – solving the transcendent equations of residual temperatures under the approximations:**
- non-energy dependent level density parameters of initial and residual fragments
- analytical expression of $\sigma_c(\varepsilon)$ (approximating $\sigma_c(\varepsilon)$ provided by OM calculations)
have provided results of prompt emission quantities, e.g. ν(A), ν(TKE), Eγ(A) etc. of $^{235}U(n_{th},f)$, $^{239}Pu(n_{th},f)$, $^{252}Cf(SF)$ in good agreement with the experimental data, validating this modeling.

- **The P(T) distributions for HF, LF and all FF resulting from these calculations allowed to obtain a new general parameterization of P(T) (preliminary)**

The global treatment of sequential emission by a P(T) distribution, employed in deterministic prompt emission models (e.g. LA, PbP), can be improved by the use of a new parameterization of P(T).

In progress:

- **To refine the parameterization of P(T) based on the present results of sequential emission calculations done for $^{235}U(n_{th},f)$, $^{239}Pu(n_{th},f)$, $^{252}Cf(SF)$**

- **Sequential emission calculations for other fissioning nuclei at higher energies e.g. $^{238}U(n,f)$, $^{237}Np(n,f)$, $^{234}U(n,f)$ at En up to about 5 MeV**

in order to provide a better general parameterization of P(T) and to study a possible variation of P(T) with energy

In the future:

Solving the transcendent equations of residual temperatures using:

- other prescriptions for the level density parameter of initial and residual fragments

- other analytical expressions of $\sigma_c(\varepsilon)$ which approximates better the $\sigma_c(\varepsilon)$ provided by optical model calculations (with optical potential parameterizations appropriate for nuclei appearing as fission fragments).

THANKS FOR YOUR ATTENTION !