Overview of two deterministic modelings for prompt emission in fission

(developed at the University of Bucharest)

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Content of the presentation

About the models (basic features)

- Similarities of the modelings
- Main difference in principle between these modelings
- Other differences concerning the prescriptions for $\sigma_c(\varepsilon)$ and the level density parameters of fragments
- Primary results of both models (multi-parametric matrices)
- Secondary results of both models
- Input parameters of the models

Model validation

- Main (primary) validation of the model itself and secondary validation of the model together with Y(A,TKE)
- Examples of main validation – matrices ν(A,TKE) and Eγ(A,TKE)
- Examples of secondary validations – prompt neutron quantities and prompt γ-ray quantities (a part of them not reported up to now)
- Examples of correlations between the prompt neutron multiplicity and prompt γ-ray quantities (e.g. Eγ, Nγ, σγ).

Prompt γ-ray spectrum provided by a global treatment based on a the distribution of prompt γ-ray energy per quanta.

The PbP model

- first publication about this model – 2005 (Nucl.Phys.A)
- previously (starting from 1998-1999) the PbP treatment was used to obtain average values of input model parameters for the LA model (most probable fragmentation approach) and for the multi-modal fission, i.e. average values corresponding to each fission mode, associated to the most probable fragmentation of each mode.
- the systematic of LA model parameters (2009) is also based on the PbP treatment.
- the emphasize of global and local even-odd effects in prompt emission (2014-2016)
- prediction of ν(A) at high En (multiple fission chances involved) (PRC 2016, NSE 2018)

A comprehensive overview of the PbP model – (2017) Eur. Phys. J. A 53, art. 159

The sequential emission modeling

- it was initially developed having as goal (objective) to obtain a general form of the residual temperature distribution P(T) – first mention in 2017 (Theory-4)
- a detailed model description – 2018, Eur. Phys. J. A 54, art. 87
- it was applied to 49 fission cases (including SF, (n_{th},f) and (n,f) at En below the threshold of the second chance fission) benefiting of reliable experimental data of Y(A,TKE) (the majority measured during the time at JRC-Geel).

This fact allowed to emphasize:

- interesting systematic behaviours of different residual quantities
- the determination of residual temperature distributions for each emission sequence $P_k(T)$ and the inclusion of sequential emission into the LA model (ND-2019).

I. SIMILARITIES

1) the same fragmentation range deterministically constructed:

- a mass range A going from symmetric fission up to a very asymmetric split

- 3 or 5 charge numbers Z taken at each A – as the nearest integer values above and below the most probable charge $Zp(A) = Z_{UCD}(A) + \Delta Z(A)$

- a large TKE range (e.g. 100 – 200 MeV) with a step size of 2 MeV or 5 MeV is taken for each fragmentation

- isobaric charge distribution p(Z,A) – Gaussian function centered on Zp(A) with ΔZ and rms as a function of A, or considering the mean values for all A, i.e.
$\Delta Z = |0.5|$ (+ for LF, - for HF) and rms = 0.6

2) the same TXE partition based on modeling at scission:

- extra-deformation energy of initial fragments at scission with respect to the full acceleration $\Delta E_{def}^{(L,H)}$

- partition of available excitation energy at scission $E_{sc}=TXE-(\Delta E_{def}^{(L)}+ \Delta E_{def}^{(H)})$ between the complementary nascent fragments under the assumptions:
i) statistical equilibrium at scission, ii) level densities in the Fermi-gas regime
i.e. $E_{sc}^{(L)}/E_{sc}^{(H)} = a_{sc}^{(L)}/a_{sc}^{(H)}$ → Note, the ratio of level density parameters remains almost the same irrespective of the prescription concerning the level density.

- the fragment excitation energy at full acceleration $E^*_{L,H} = \Delta E_{def}^{(L,H)} + E_{sc}^{(L,H)}$

II. MAIN DIFFERENCE IN PRINCIPLE

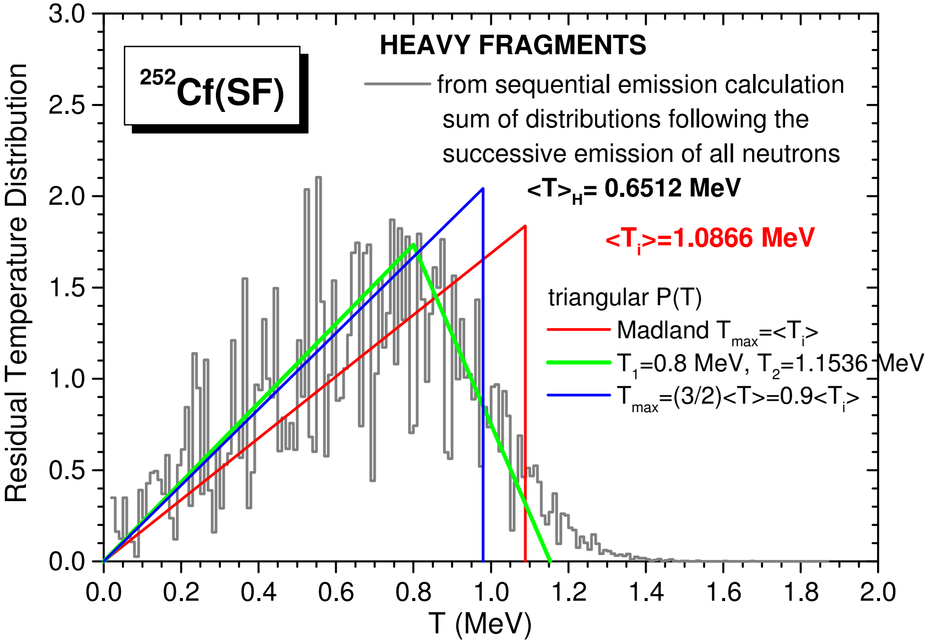
concerns the treatment of sequential emission

PbP: a global treatment of sequential emission based on the residual temperature distribution P(T) → the center-of-mass energy spectrum of prompt neutrons associated to an initial fragment {A, Z} at a given TKE value is calculated as:

$$\boxed{\Phi(\varepsilon)=\int_{0}^{T\max} P(T)\,\varphi(\varepsilon,T)\,dT}$$

The PbP computer code allows P(T) as numerical files or analytical expressions

Example →



A.Tudora, F.-J.Hambsch, V.Tobosaru, Eur.Phys.J.A 54 (2018) 87

Deterministic modeling of sequential emission:

based on recursive equations of residual temperature following the successive emission of each prompt neutron from each initial fragment {A, Z} at each TKE:

$$\boxed{\overline{E_r}^{(k-1)} - S_n^{(k-1)} - <\varepsilon>_k = a_k T_k^{\,2}}$$

For k=0 (initial fragments) $<E_r>^{(0)}$ is E* at full acceleration resulting from the TXE partition

III. Other differences concern the prescriptions for $\sigma_c(\varepsilon)$ and the level density parameters, i.e.

PbP:

- **$\sigma_c(\varepsilon)$: i) OM calculation with parameterizations adequate for nuclei appearing as FF (e.g. B-G, K-D etc.), ii) analytical expressions, iii) constant**
- **level density parameter: energy-dependent (superfluid model), non-energy dependent (e.g. systematics of Egidy-Bucurescu, of Gilbert-Cameron etc.)**

Sequential emission modeling:

- **$\sigma_c(\varepsilon)$: i) analytical expressions, ii) constant**
- **level density parameter: non-energy dependent (e.g. systematic of Egidy-Bucurescu for BSFG, or of Gilbert-Cameron etc.)**

IV. PRIMARY RESULTS of both modelings are

multi-parametric matrices of different quantities q(A,Z,TKE) characterizing the fragments and the prompt emission

e.g. E*(A,Z,TKE), ν(A,Z,TKE), <ε>(A,Z,TKE), Eγ(A,Z,TKE), Φ(ε,A,Z,TKE) etc.
generically labeled *q(A,Z,TKE)*

PbP → *q(A,Z,TKE)* (is a global value corresponding to all emission sequences)

Sequential emission → $q_k(A,Z,TKE)$, k = 1, … *n(A,Z,TKE)* (number of seq.)

$$\boxed{\bar{q}(A,Z,TKE) = \frac{1}{n(A,Z,TKE)} \sum_{k=1}^{n(A,Z,TKE)} q_k(A,Z,TKE)}$$

V. SECONDARY RESULTS of both modelings

Consist of single distributions of different quantities (*q(A), q(TKE), q(Z)*) and total average quantities (<*q*>). They are obtained by averaging the primary results q(A,Z,TKE) over fission fragment distributions.

$$Y(A,Z,TKE) = p(Z,A)\; Y_{exp}(A,TKE)$$

Y(A,TKE) are needed (as input). Experimental Y(A,TKE) data are preferred.
Y(A,TKE) can be also reconstructed from experimental data Y(A), TKE(A), σ_{TKE}(A)

VI. INPUT PARAMETERS of both modelings

- **For the primary results (multi-parametric matrices) both modelings do not use free or adjustable parameters.**

  They need only data from recommended nuclear data libraries (RIPL1-3).

  E.g. mass excesses (Audi and Wapstra), β2 deformations (e.g. Möller and Nix), shell corrections (e.g. Möller and Nix, Myers and Swiatecki), optical model parameterizations (segment IV of RIPL), etc.

  This fact assures the possibility of prediction.

- **For other results (i.e. average quantities as a fct. of A, of TKE, total average quantities)**

  Y(A,TKE) data are needed as input.

VII. MODEL VALIDATION

- **Main (primary) validation, of the prompt emission model itself**

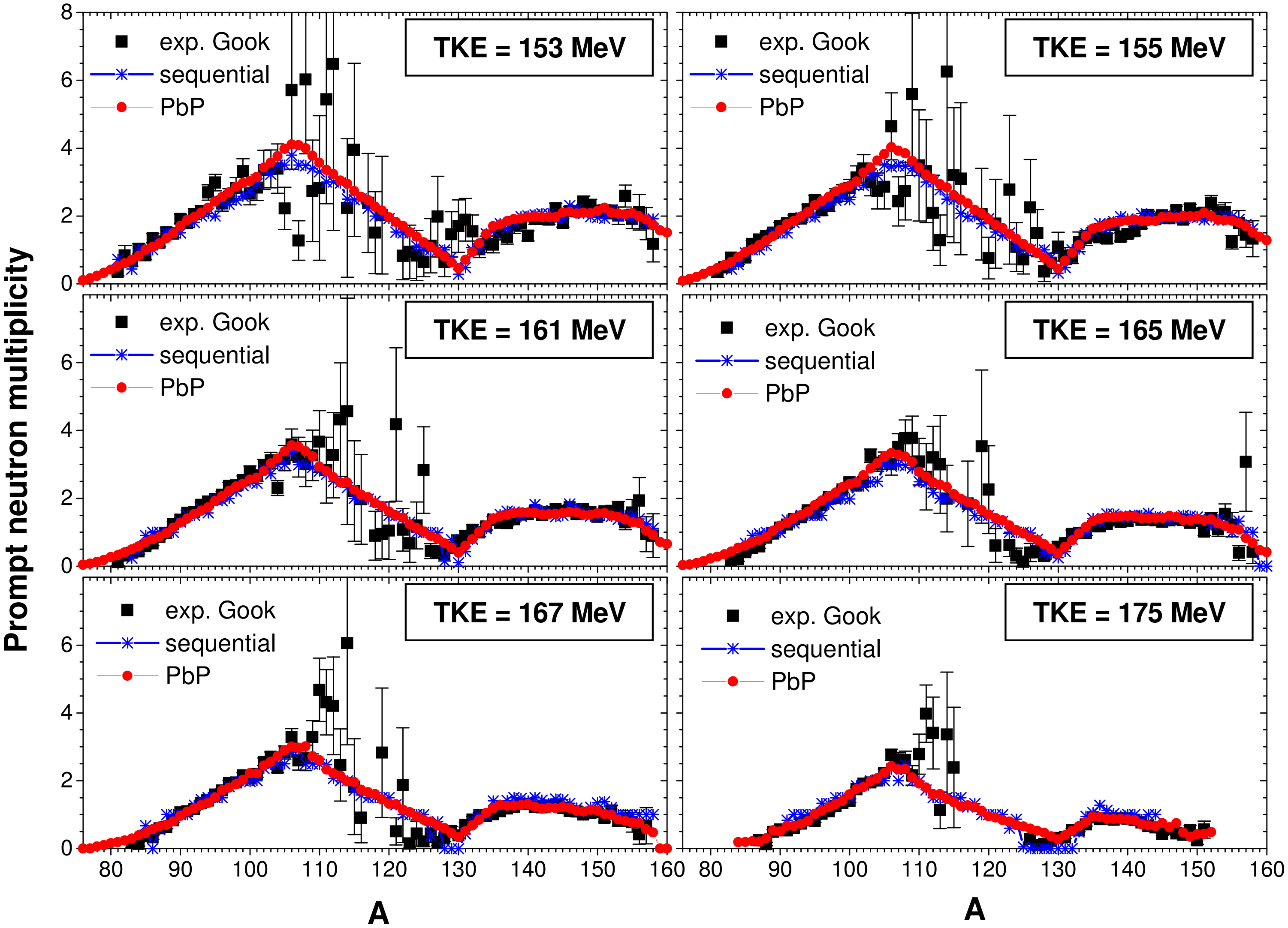
Consists of the comparison of primary model results (multi-parametric matrices) with experimental data, e.g. ν(A,TKE), Eγ(A,TKE).

- **Secondary validation, of the prompt emission model together with Y(A,TKE)**

Consists of the comparison of different single distributions and/or total average quantities with experimental data, e.g. ν(A), ν(TKE), <ε>(A), <ε>(TKE), Eγ(A), Eγ(TKE), Nγ(A), Nγ(TKE), Φ(ε), N(E) (PFNS), Sγ(Eγ) (PFGS), <ν>, <Eγ>, <Nγ> etc.

Example of main (primary) validation

The ν(A,TKE) matrix of ^{235}U(n$_{th}$,f) in the 2D representation of ν(A) at a given TKE value



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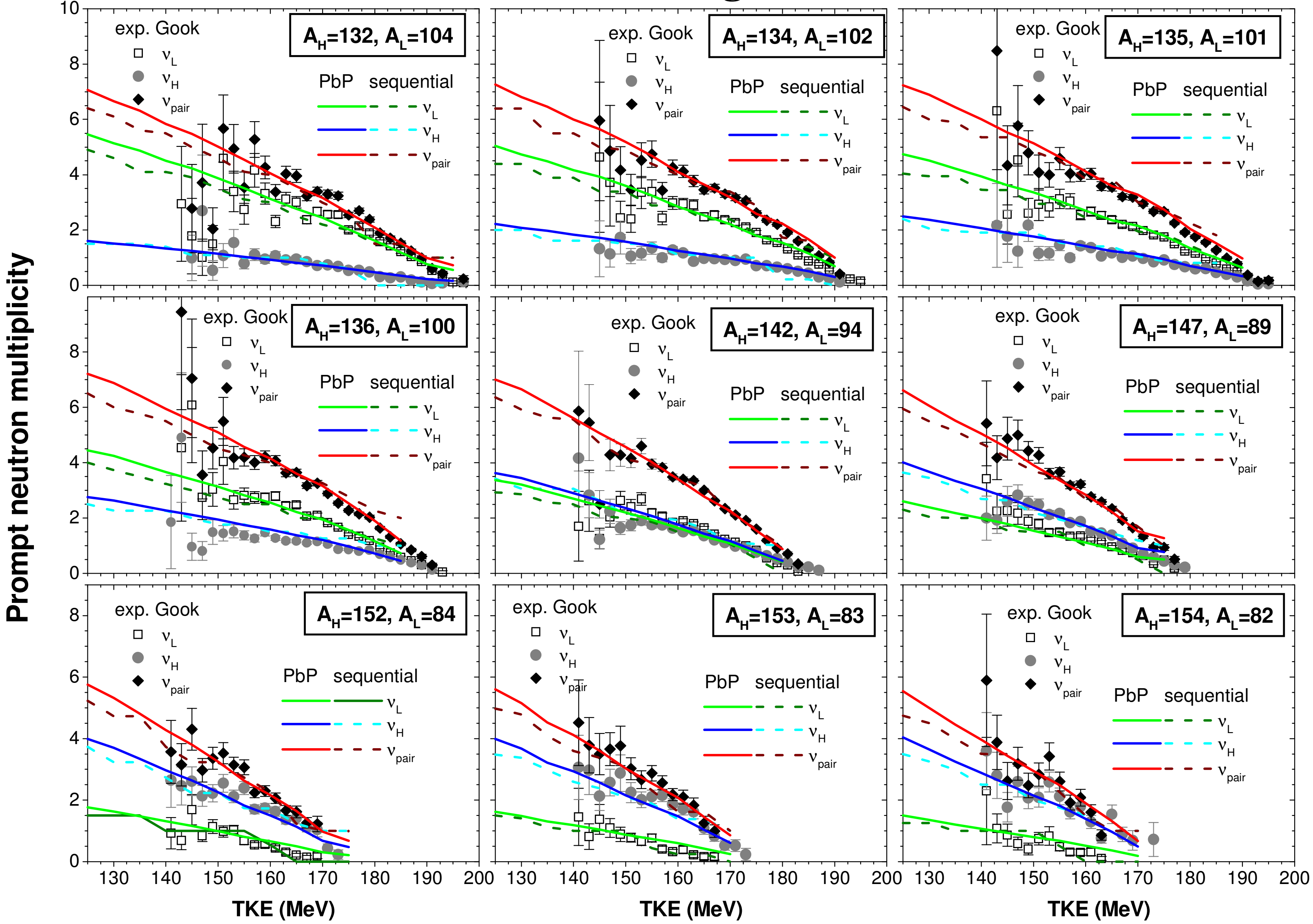
Example of main (primary) validation

The ν(A,TKE) matrix of ^{235}U(n_{th},f) in the 2D representation of ν(TKE) for a fragment mass A



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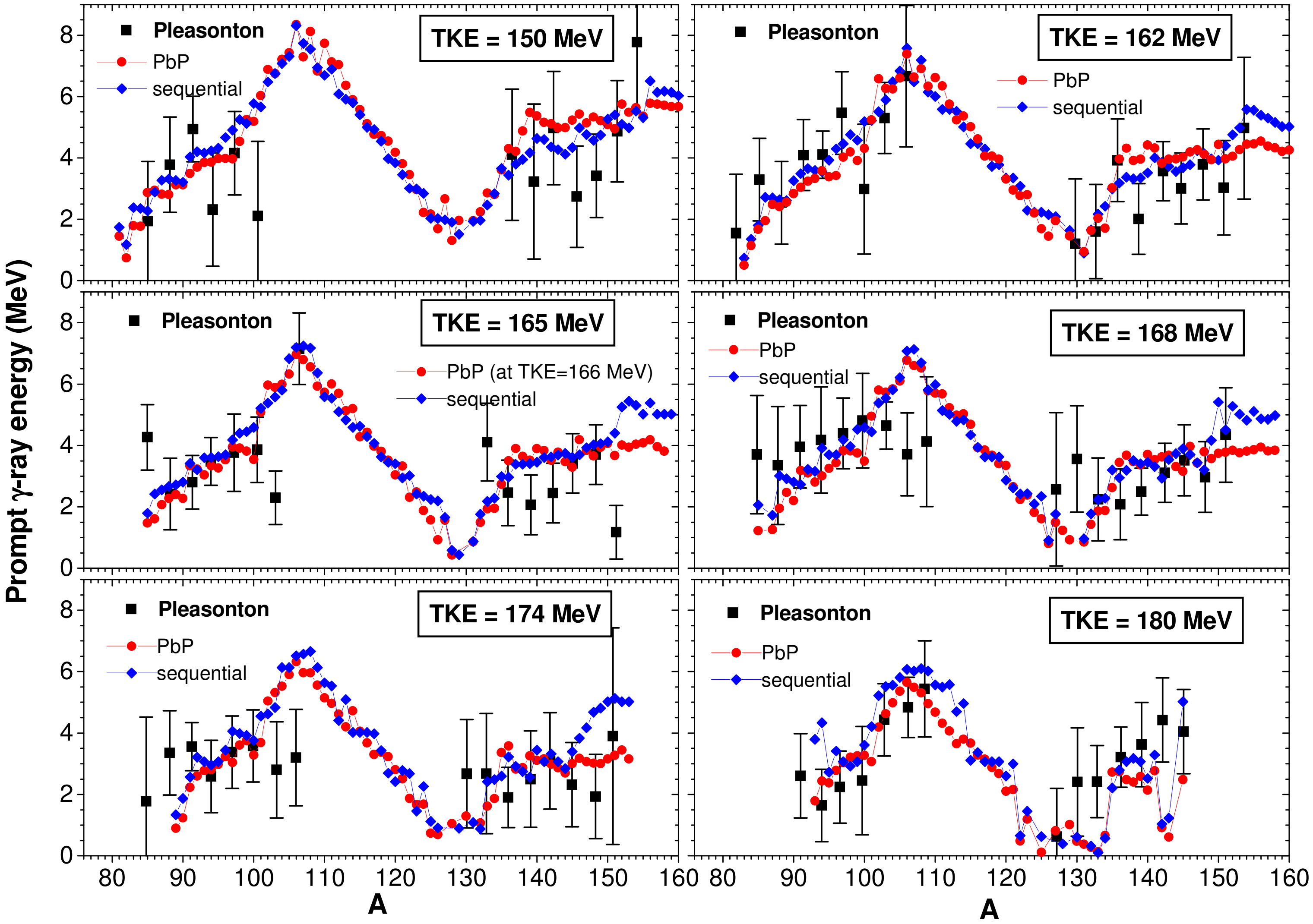
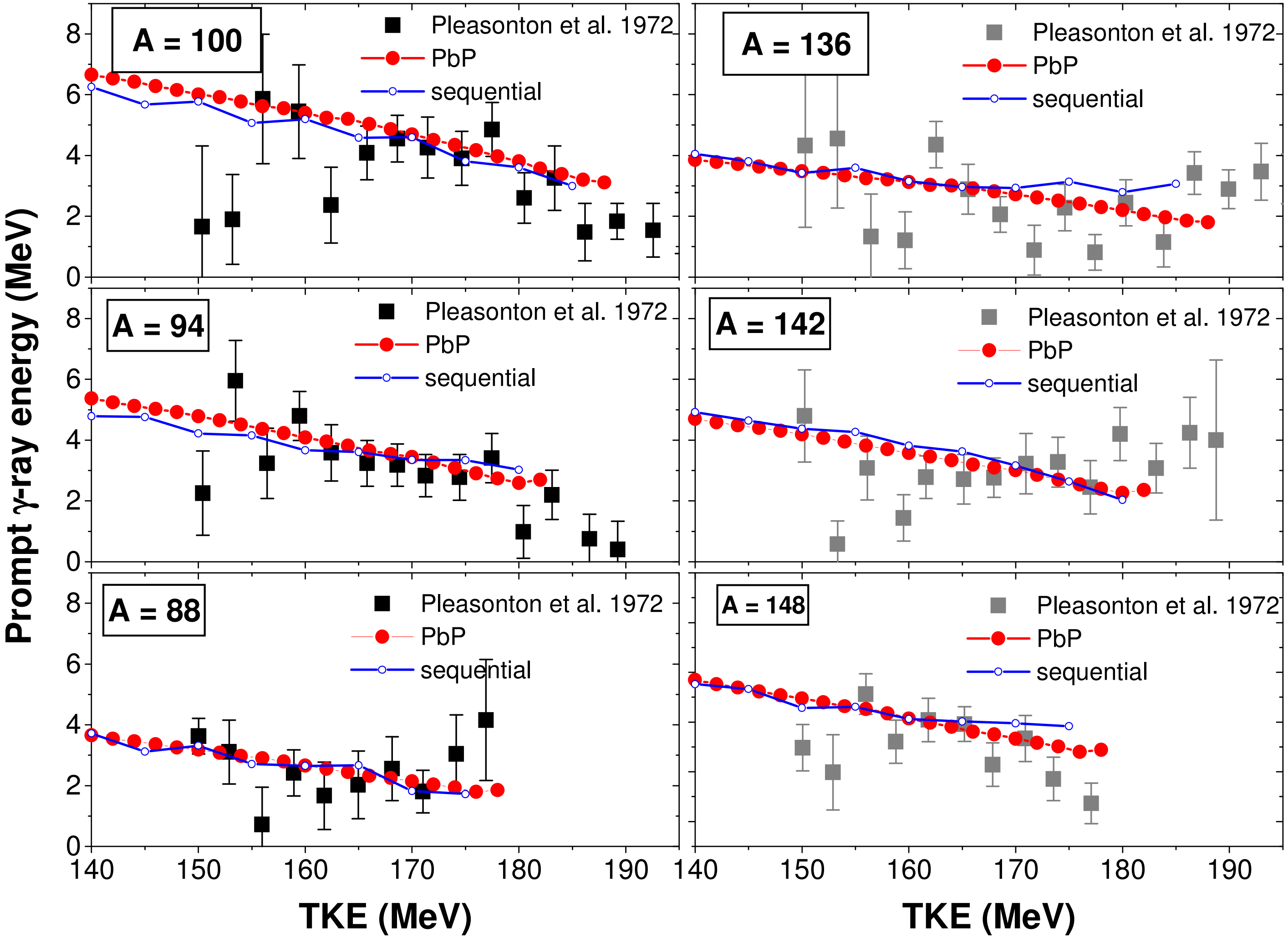
Example of main (primary) validation – not reported up to now

Prompt γ-ray energy matrix Eγ(A,TKE) $^{235}U(n_{th},f)$ in the 2D representation of Eγ(A) at a given TKE value

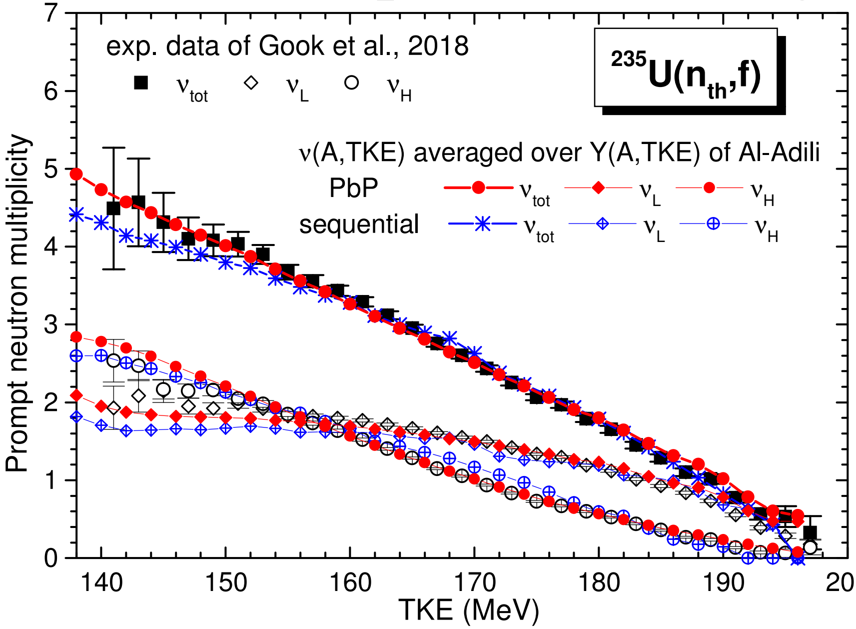


Example of main (primary) validation – not reported up to now

Prompt γ-ray energy matrix Eγ(A,TKE) ^{235}U(n$_{th}$,f) in the 2D representation of Eγ(TKE) for a given fragment mass



Examples of secondary validations – prompt neutrons



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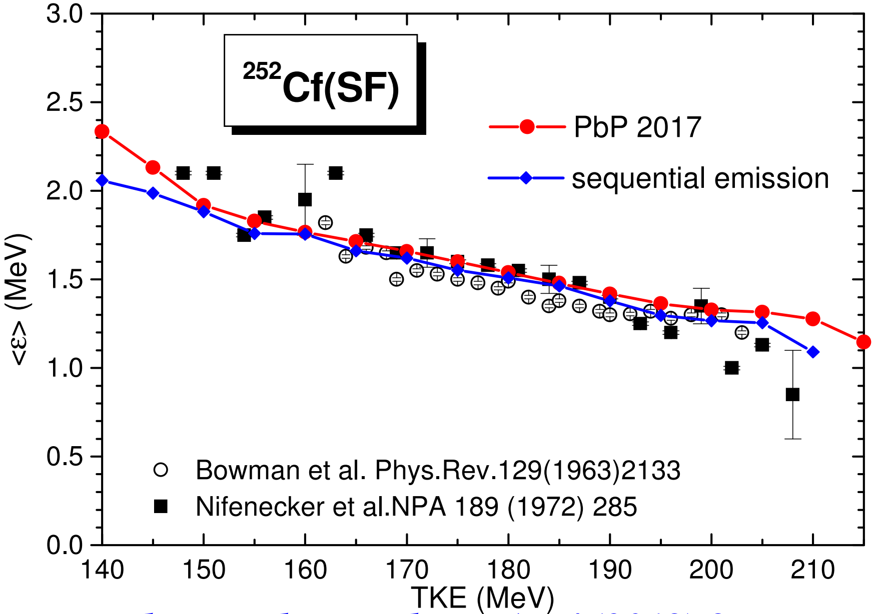

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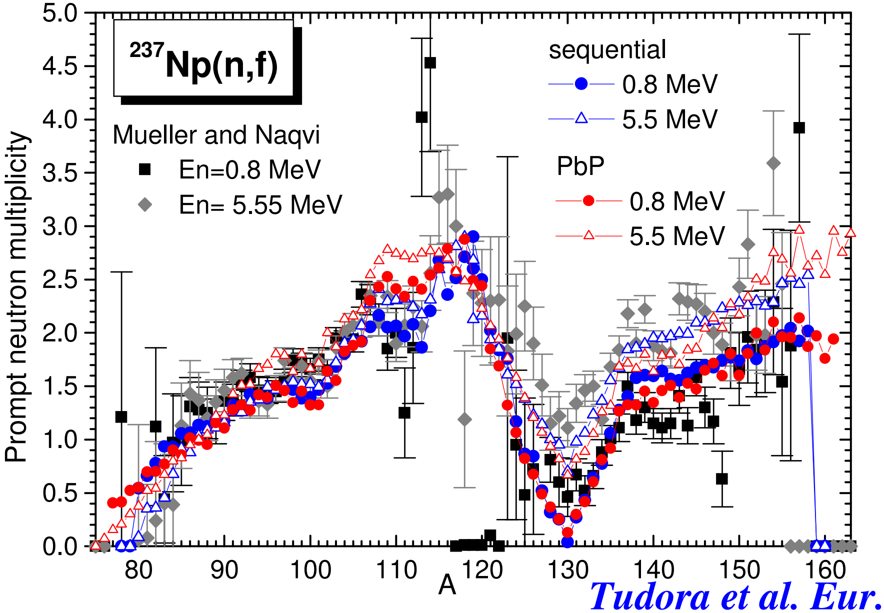

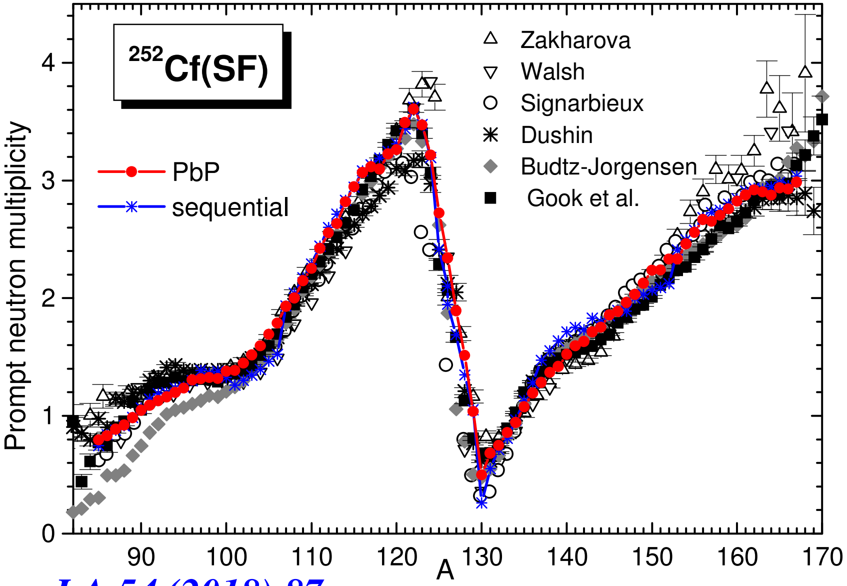

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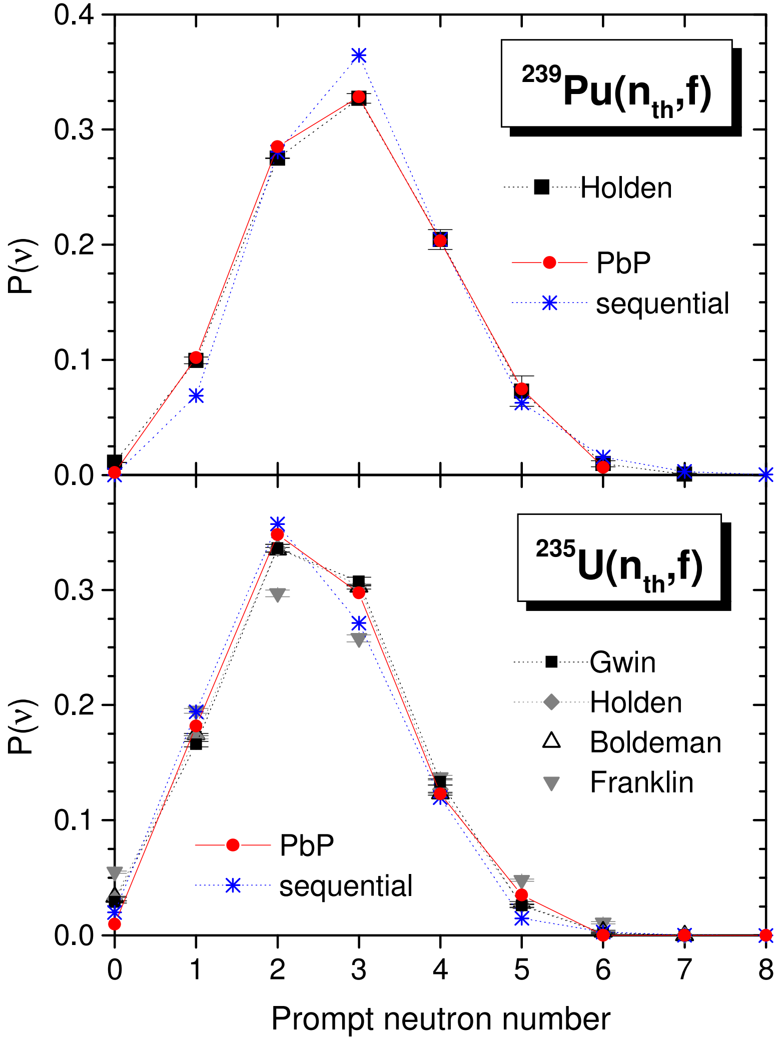
Examples of secondary validations – prompt neutrons

Prompt neutron distribution



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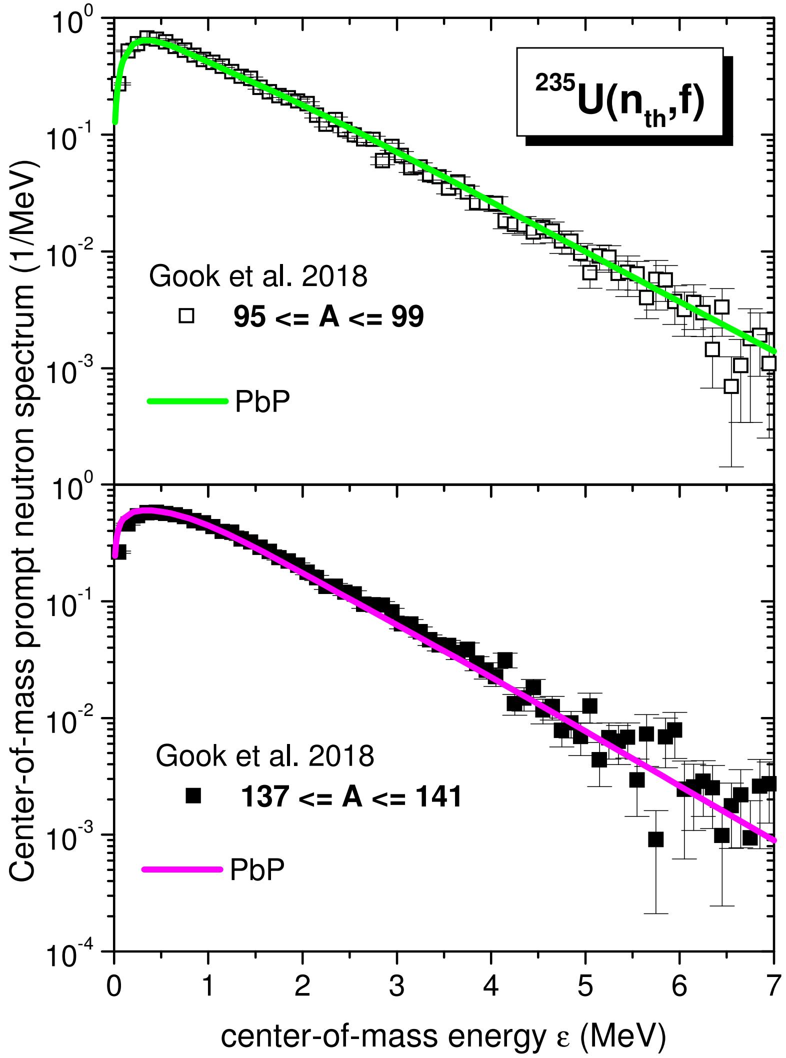
Prompt neutron spectrum in the CMS



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Prompt γ-ray results not reported up to now

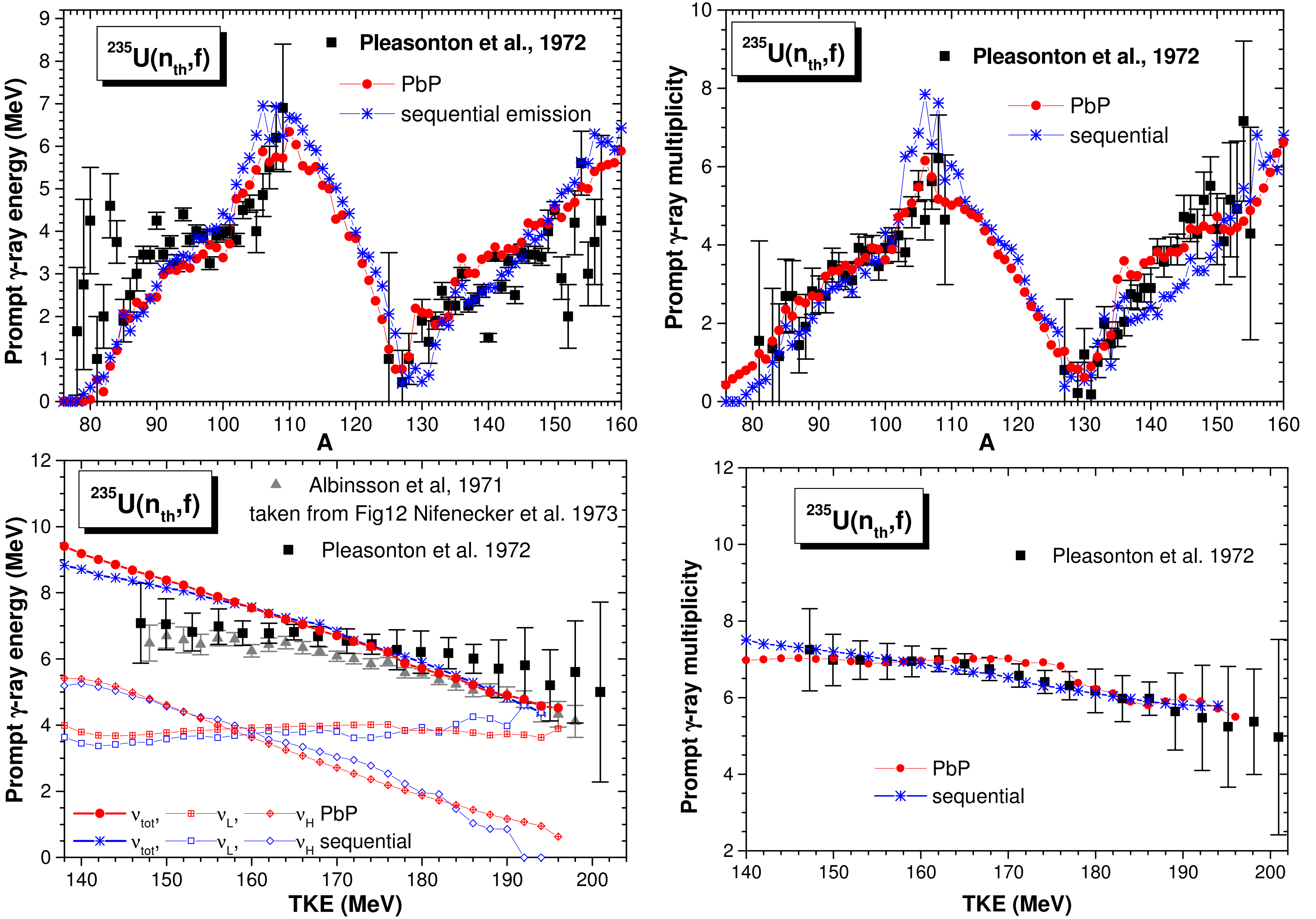
- **different prompt γ-ray quantities**
obtained by averaging the corresponding multi-parametric matrices over experimental fragment distributions

- **correlations of prompt γ-ray quantities with the prompt neutron multiplicity**

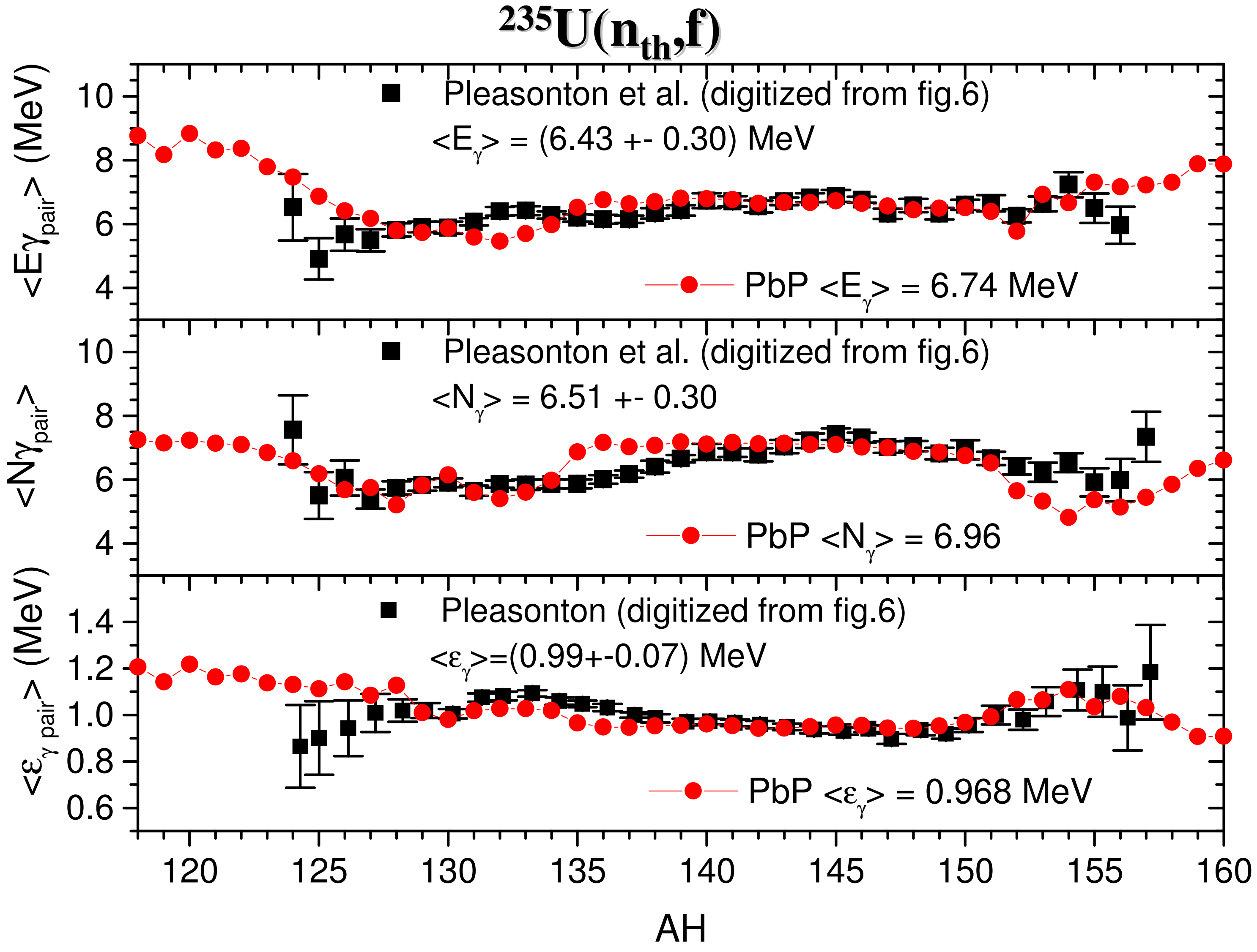
The following experimental Y(A,TKE) distributions measured at JRC-Geel were used:

$^{235}U(n_{th},f)$ (Al-Adili et al., 2012, 2016)
$^{239}Pu(n_{th},f)$ (Wagemans et al., 1984)
$^{252}Cf(SF)$ (Göök et al., 2014)

Average prompt γ-ray energy and multiplicity of ^{235}U(n_{th},f) as a function of A and as a function of TKE



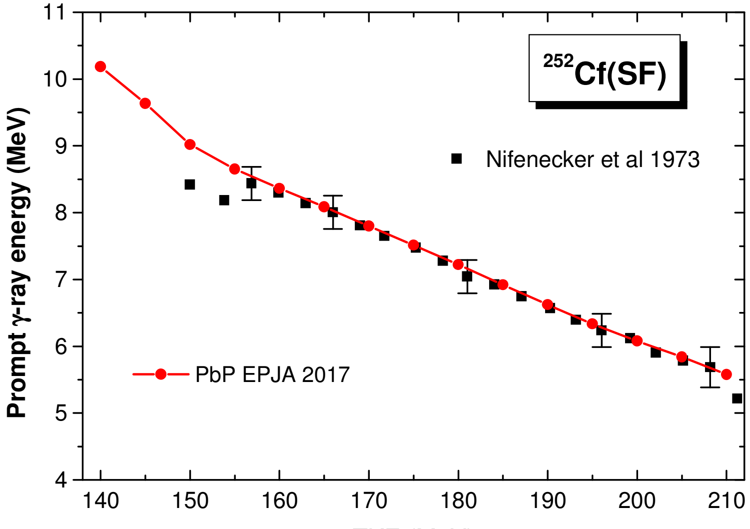
Average prompt γ-ray energy, multiplicity and energy per quanta of fragment mass pair

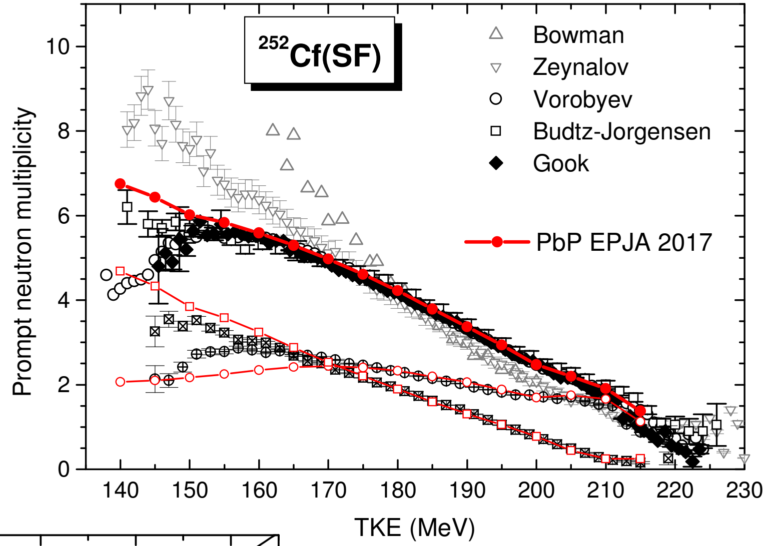


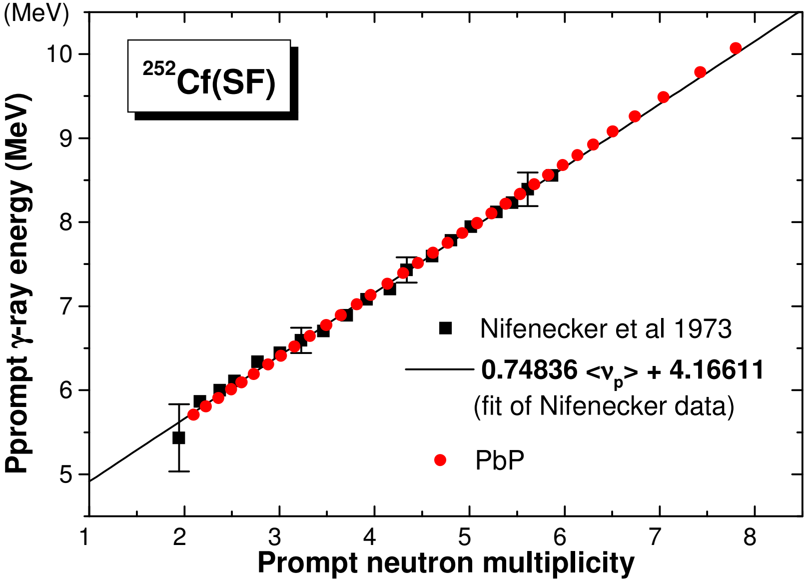
Prompt γ-ray quantities in correlation with the prompt neutron multiplicity

The linear correlation between Eγ and ν and Nγ and ν is obvious if the experimental data and/or the model results of Eγ(TKE), Nγ(TKE) and ν(TKE) exhibit linear decreases.

example for ^{252}Cf(SF)



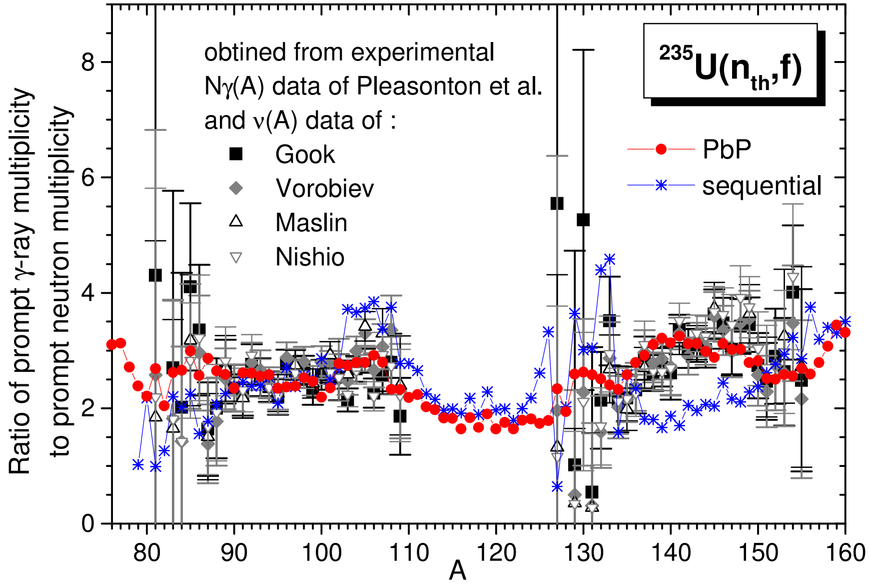


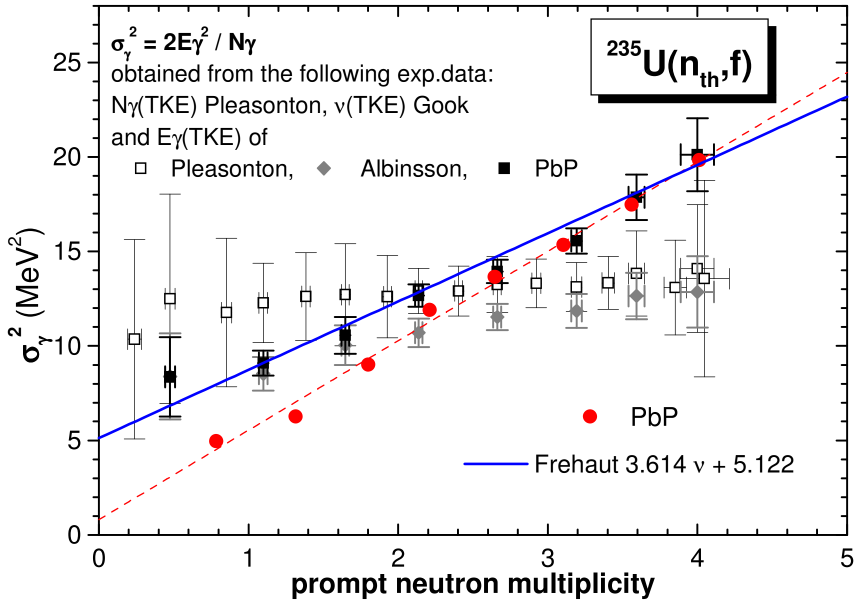


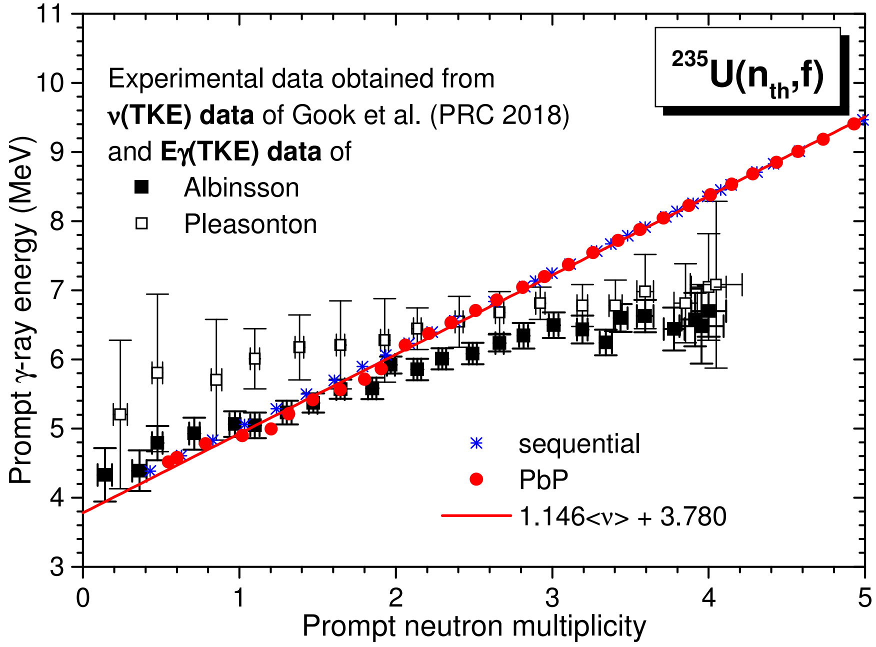
🡐Linear correlation between the prompt γ-ray energy and the prompt neutron multiplicity obtained from the calculated <ν>(TKE) and <Eγ>(TKE).

Prompt γ-ray quantities in correlation with the prompt neutron multiplicity

example for ^{235}U(n$_{th}$,f)







$$\sigma_\gamma^2 = < E_\gamma^2 > - < E_\gamma >^2 = \varepsilon_\gamma^2 (\overline{N_\gamma^2} + \overline{N_\gamma} - \overline{N_\gamma}^2) =$$

$$\sigma_\gamma^2 = \varepsilon_\gamma^2 (\sigma_{N_\gamma}^2 + \overline{N_\gamma})$$

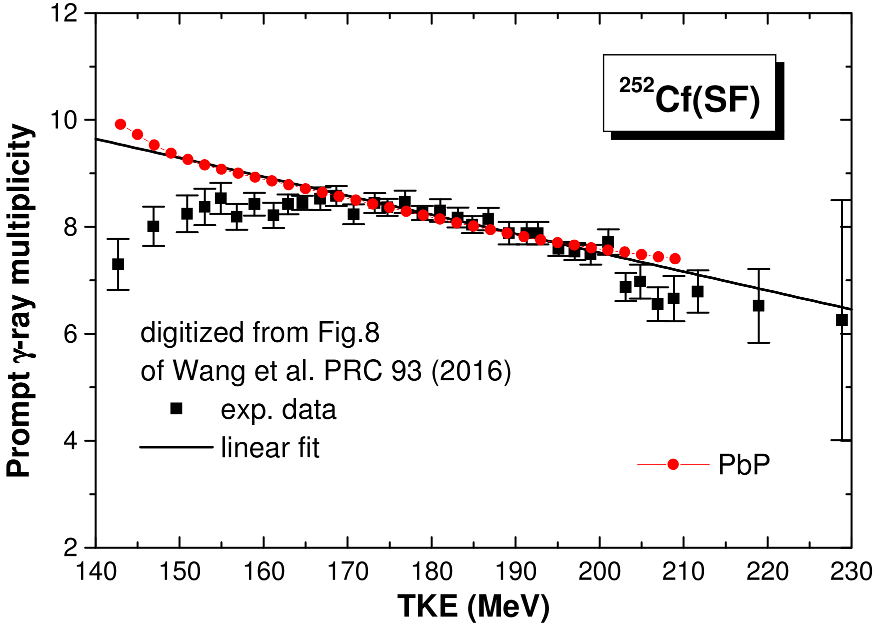
Poisson $\sigma_{N_\gamma}^2 = \overline{N_\gamma}$

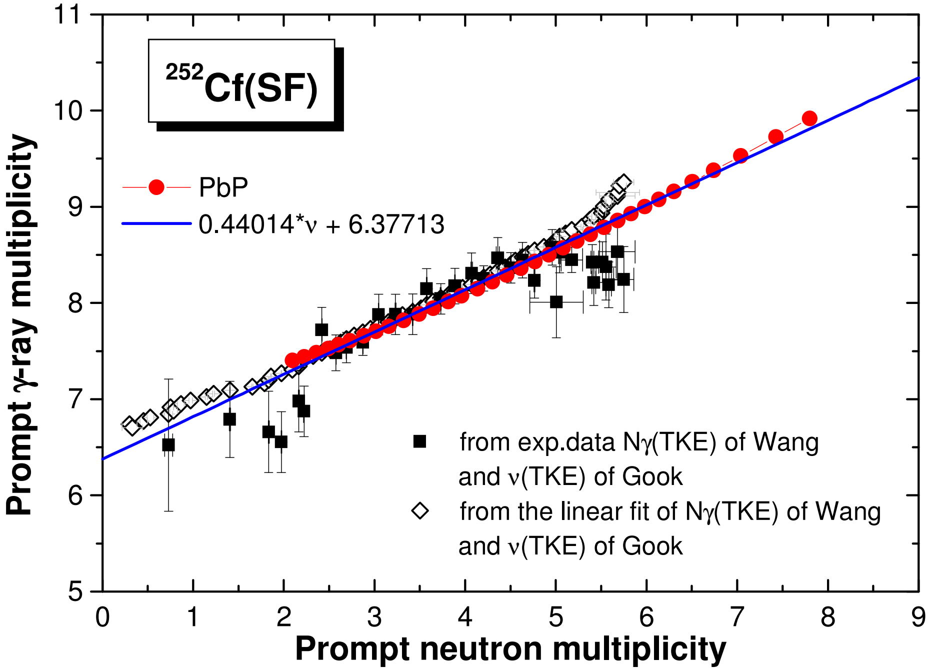
$$\sigma_\gamma^2 = 2\varepsilon_\gamma^2 \overline{N_\gamma} = 2 < E_\gamma >^2 \Big/ \overline{N_\gamma}$$

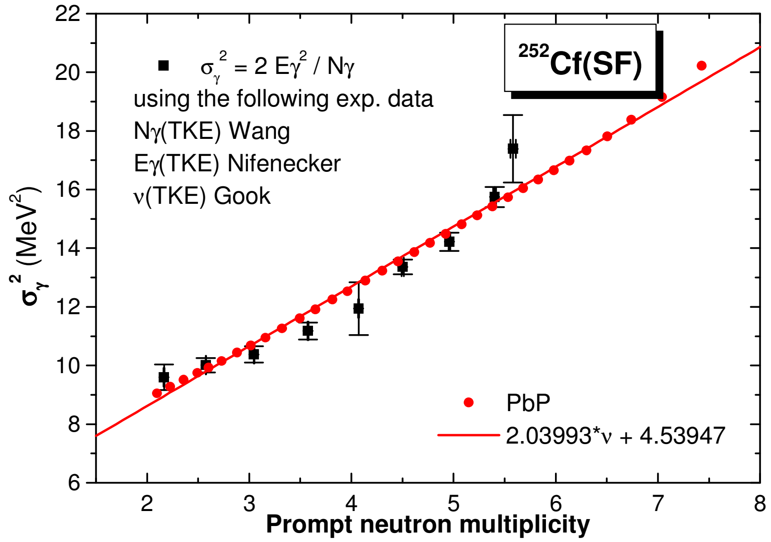
🡐Linear correlation between the prompt γ-ray energy and the prompt neutron multiplicity obtained from the calculated <ν>(TKE) and <Eγ>(TKE).

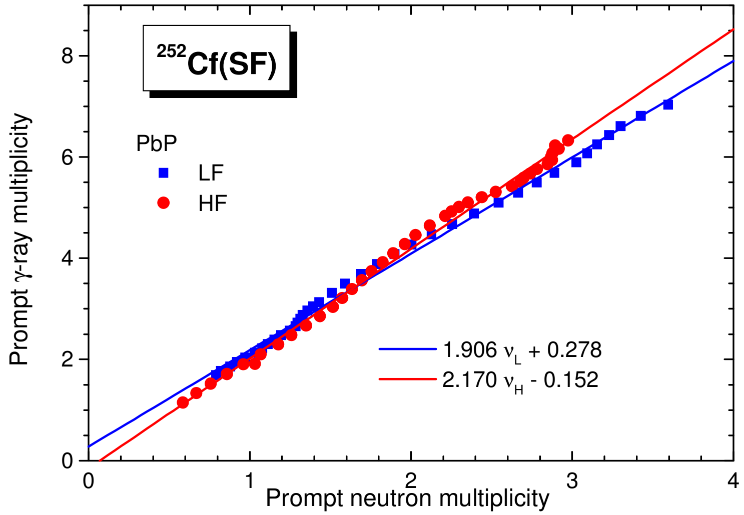
Prompt γ-ray quantities in correlation with the prompt neutron multiplicity

example for ^{252}Cf(SF)

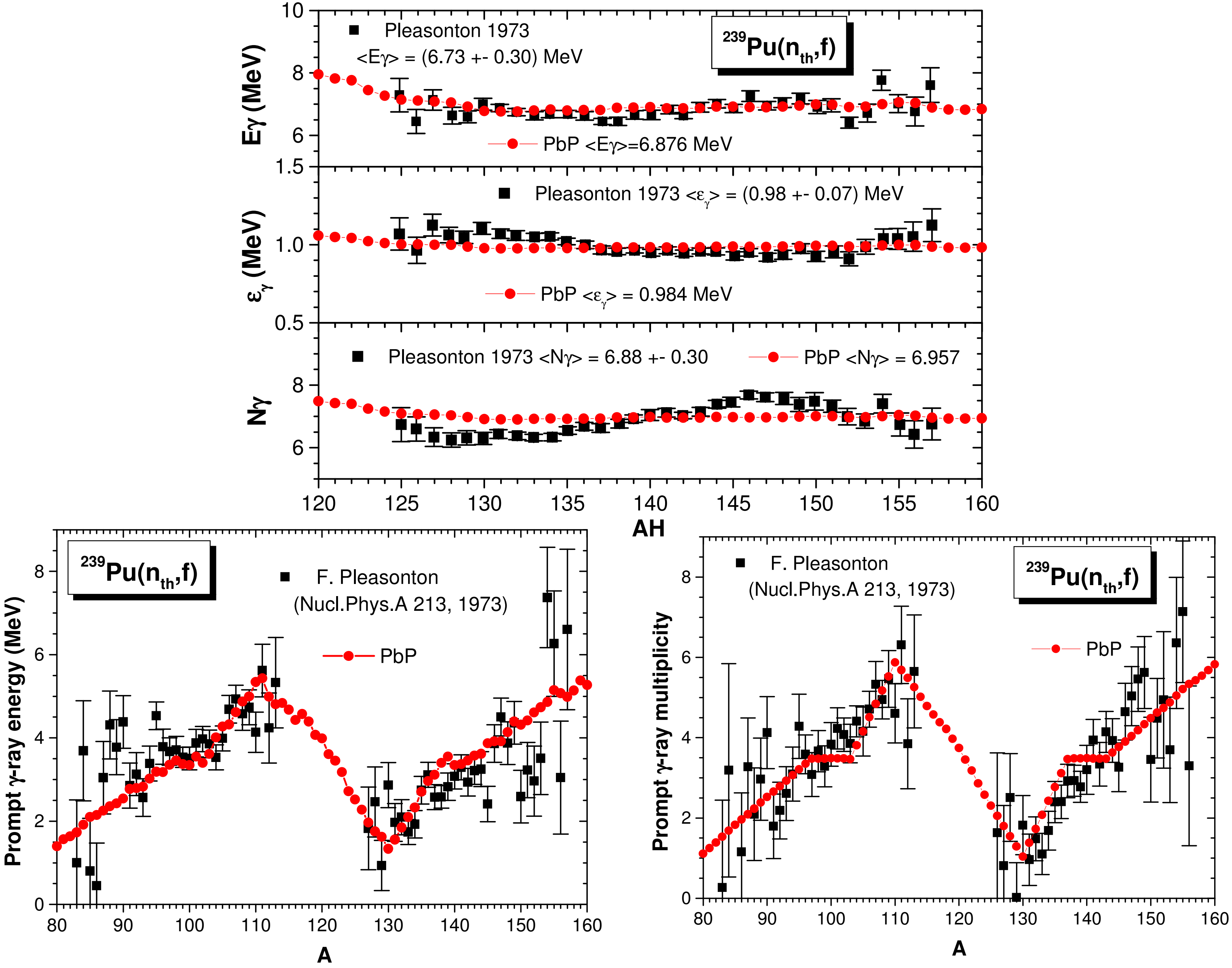




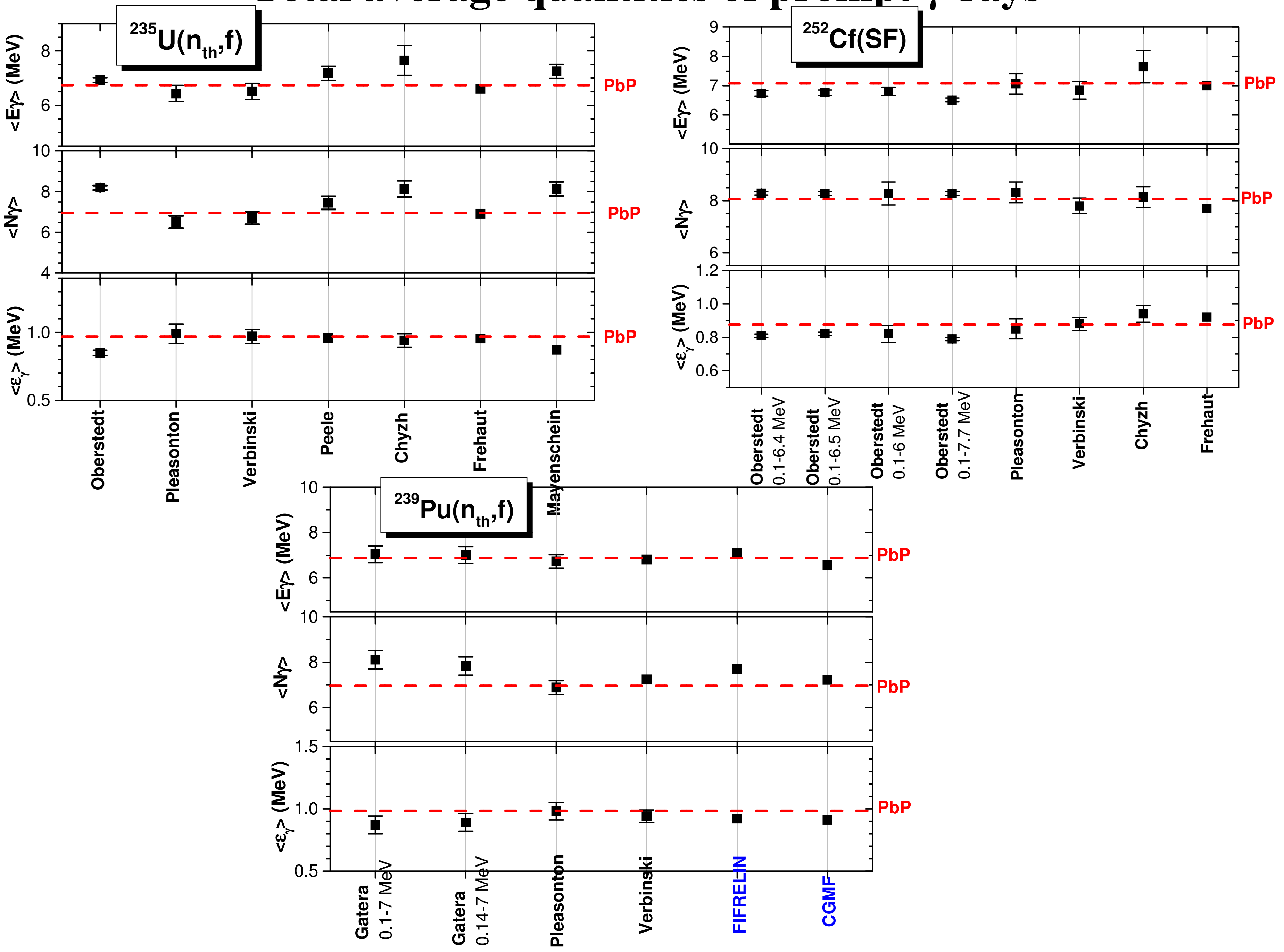




Prompt γ-ray results for ^{239}Pu(n_{th},f)



Total average quantities of prompt γ-rays



Prompt γ-ray spectrum

- **As an alternative to the statistical H-F calculation of γ-ray emission in competition with neutron emission from many nuclei appearing as fission fragments (great part of these nuclei having scarce or unknown level schemes) requiring a long computing time → we propose**

- **a global treatment based on an idea which is similar to the one used for prompt neutron emission. This global treatment is based on the distribution of prompt γ-ray energy per quanta D(εγ) of the light and heavy fragment groups**

- **The prompt γ-ray spectrum is obtained by integration of the spectrum for a given energy per quanta over the D(εγ) distribution separately for LF and HF and the multiplication of each integral with the average prompt γ-ray multiplicity of to the light and heavy fragment groups.**

- **This is a very simple modeling – without free or adjustable parameters- adequate for evaluation purposes.**

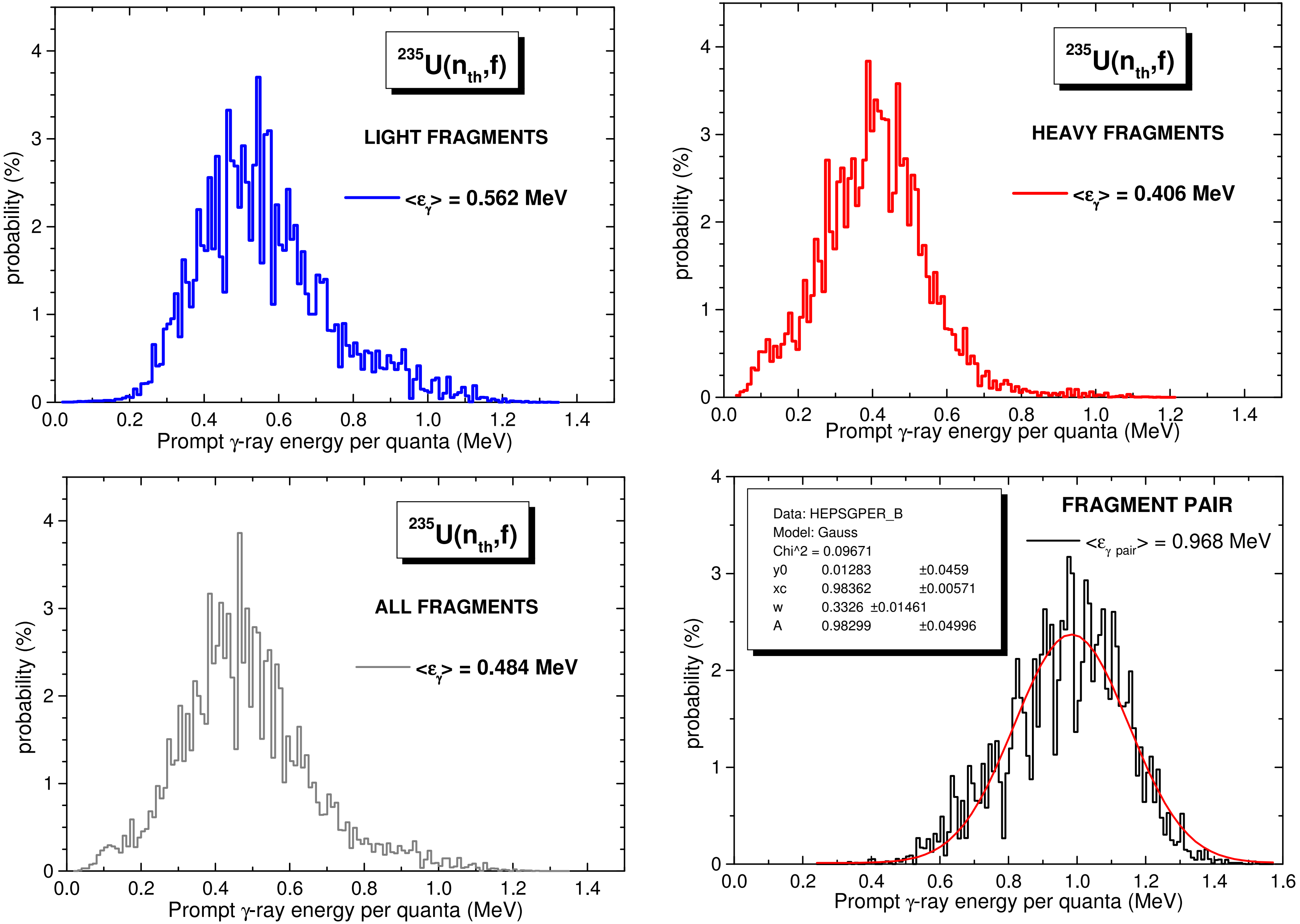
$$\boxed{\Phi(E_\gamma) = \overline{N_\gamma}_L \int_0^{\varepsilon_{\gamma\max}^{(L)}} D_L(\varepsilon_\gamma)\varphi(E_\gamma,\varepsilon_\gamma)\,d\varepsilon_\gamma + \overline{N_\gamma}_H \int_0^{\varepsilon_{\gamma\max}^{(H)}} D_H(\varepsilon_\gamma)\varphi(E_\gamma,\varepsilon_\gamma)\,d\varepsilon_\gamma}$$

The prompt γ-ray spectrum for a given energy per quanta is considered under the approximation of a constant population c.s. This is supported by the photo-absorption c.s. of the inverse process which is almost constant for Eγ up to about 7 - 8 MeV, followed by the brusque increase due to the GDR (positioned at about 14-15 MeV, according to GDR parameterizations of RIPL 1-3)

$$\varphi(E_\gamma,\varepsilon_\gamma) = \begin{cases} \dfrac{E_\gamma}{\varepsilon_\gamma^2}\exp(-E_\gamma/\varepsilon_\gamma) & E_\gamma \le 4\varepsilon_\gamma \\ \dfrac{E_\gamma^2}{4\varepsilon_\gamma^3}\exp(-E_\gamma/\varepsilon_\gamma) & E_\gamma > 4\varepsilon_\gamma \end{cases}$$

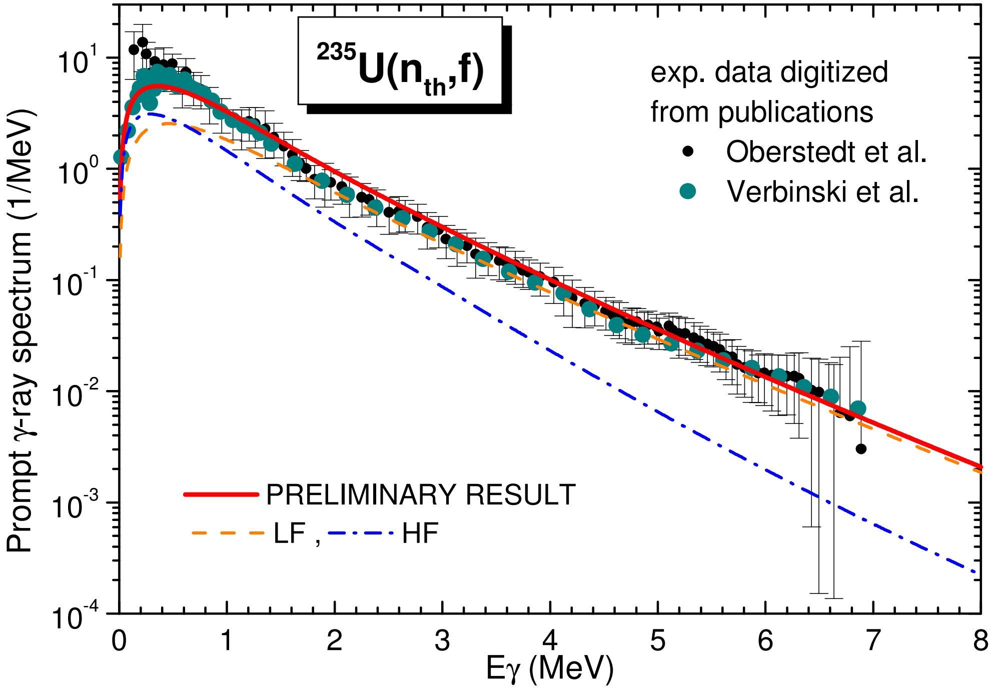
Distribution of prompt γ-ray energy per quanta D(ε_γ) for ^{235}U(n_{th},f)

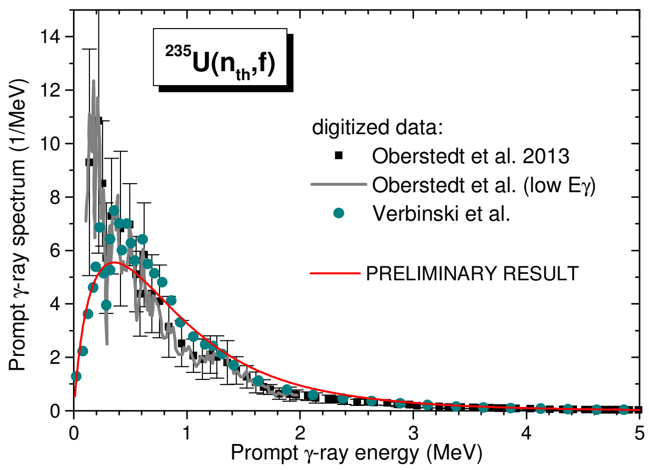
obtained from the matrix ε_γ(A,Z,TKE) of PbP and Y(A,Z,TKE)

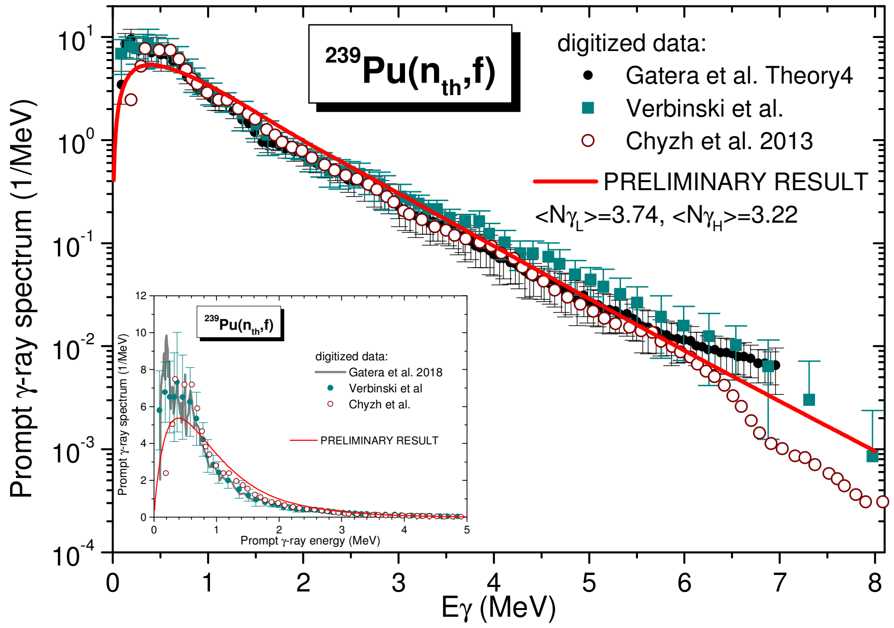


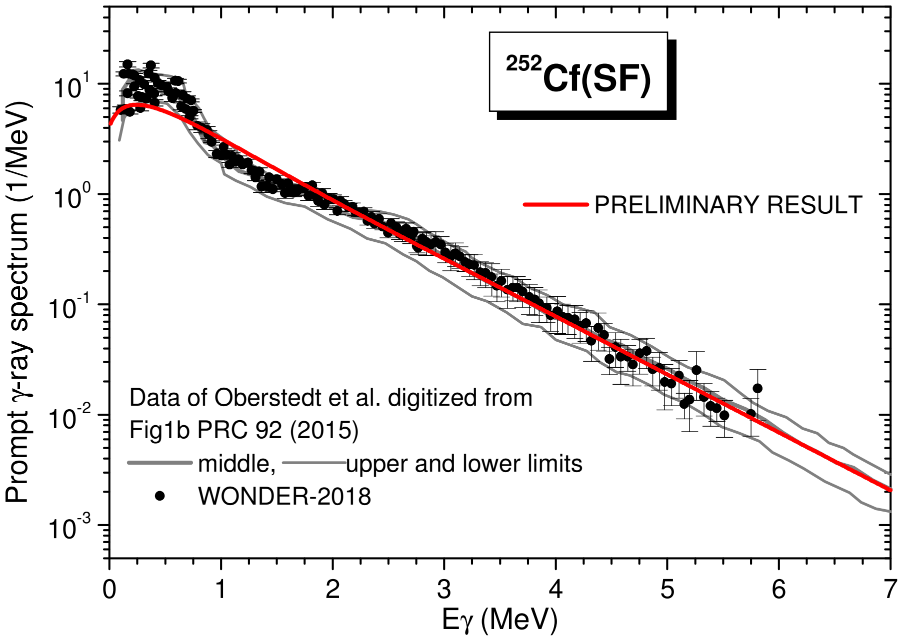
```
Data: HEPSGPER_B
Model: Gauss
Chi^2 = 0.09671
y0      0.01283         ±0.0459
xc      0.98362         ±0.00571
w       0.3326  ±0.01461
A       0.98299         ±0.04996
```

Prompt γ-ray spectra – preliminary results









CONCLUSIONS

- **Both deterministic modelings (PbP with a global treatment of sequential emission and with a detailed treatment of sequential emission) were submitted to a rigorous and detailed validation including:**
  1) **The validation of the model itself – a very good description of multi-parametric experimental data (recent ν(A,TKE) data and less recent data of Eγ(A,TKE))**
  2) **The validation of the model together with a distribution Y(A,TKE) – a very good description of experimental single distributions and total average quantities related to prompt neutrons (ν(A), ν(TKE), <ε>(A), <ε>(TKE), P(ν), <ν>, Φ(ε), N(E) etc) and to prompt γ-rays (Eγ(A), Eγ(TKE), Nγ(A), Nγ(TKE), ε_γ(A), <Eγ>, < Nγ>, <ε_γ>, etc)**

- **Correlations between the prompt neutron multiplicity and different prompt γ-ray quantities (e.g. Eγ, Nγ etc.) were emphasized, too.**

- **The preliminary results of prompt γ-ray spectrum, provided by a simple modeling including a global treatment based on the distribution of prompt γ-ray energy per quanta D(ε_γ) (provided by the PbP model), give an overall good description of the recent experimental data for ^{235}U(n_{th},f), ^{239}Pu(n_{th},f) and ^{252}Cf(SF).**

- **The determination of a general analytical form for the distribution of prompt γ -ray energy per quanta (as in the case of the triangular P(T) used for PFNS calculation) is in progress.**

Acknowledgements – a great part of this work was done in the frame of the Romanian research project PN-III-P4-PCE-2016-0014