Residual temperature distributions and systematic behaviours of residual quantities following the sequential emission of prompt neutrons

- preliminary results -

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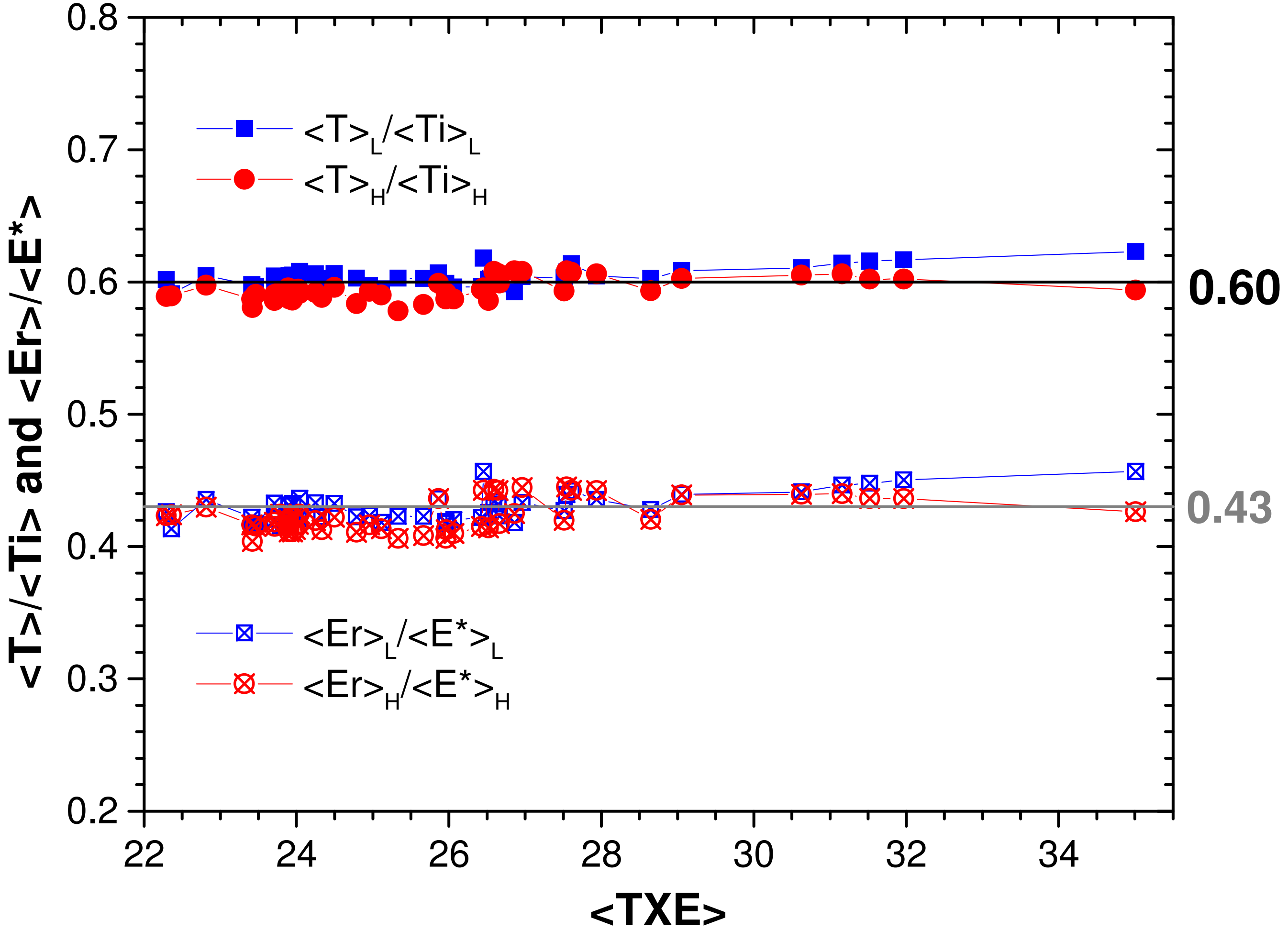
The development of a sequential emission modeling (with a deterministic treatment) has had as initial goal the determination of a general form for P(T) to be used in prompt emission models with a global treatment of the sequential emission, like PbP and LA.

For this reason the sequential emission modeling was applied to many fissioning systems benefiting of experimental Y(A,TKE) data, as follows :

- **SF: ^{252}Cf(SF), ^{236,238,240,242,244}Pu(SF)**
- **(n_{th},f): ^{235}U(n_{th},f), ^{239}Pu(n_{th},f) and ^{233}U(n_{th},f)**
- **(n,f) below the threshold of the second chance fission:**
  ^{237}Np(n,f) at 12 En going from 0.3 and 5.5 MeV
  ^{238}U(n,f) at 14 En going up to 5.5 MeV
  ^{234}U(n,f) at 14 En ranging from 0.2 to 5 MeV

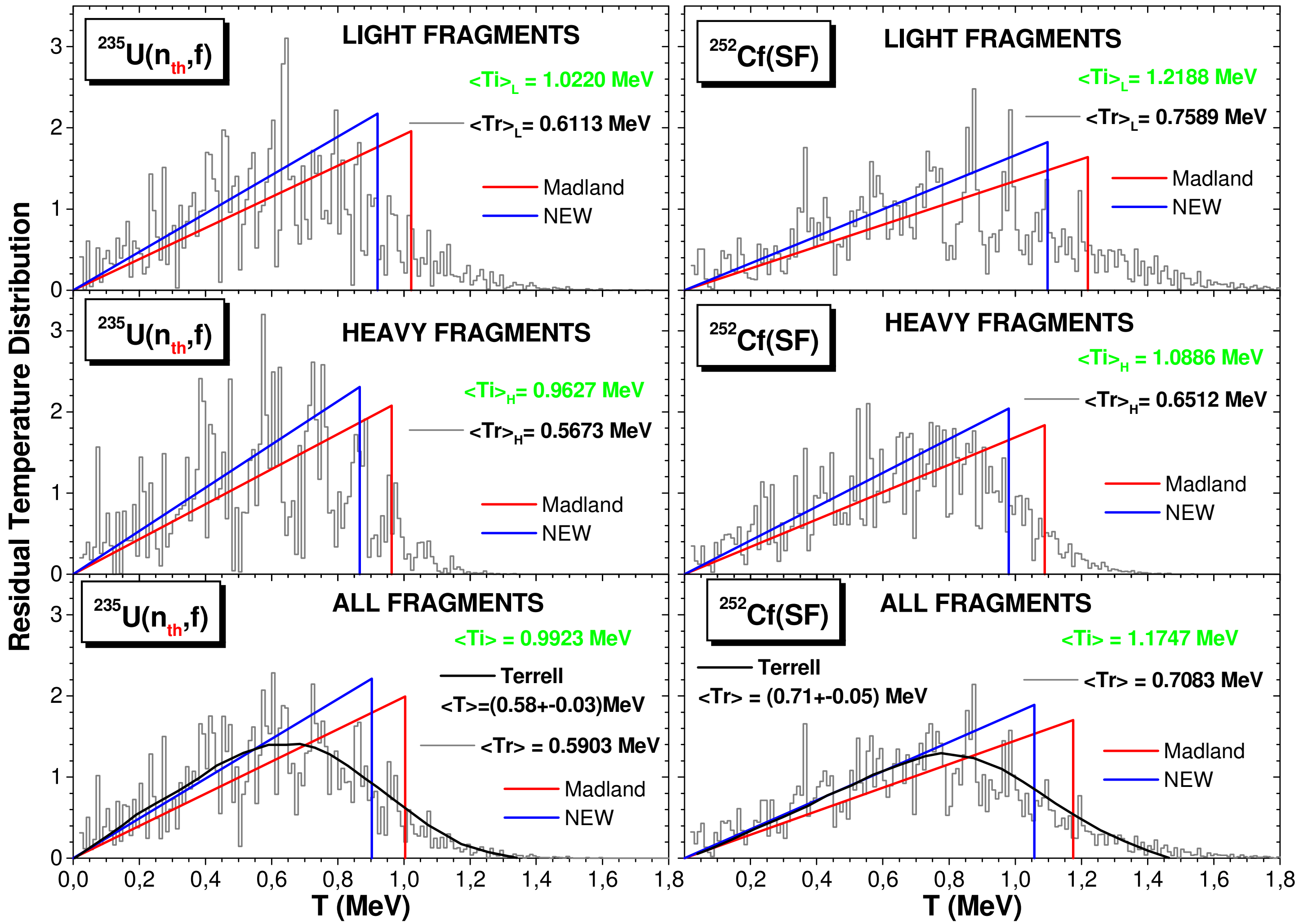
i.e. a total number of 49 fission cases covering a large range of nuclei and TXE values. These allowed to determine interesting systematics.

The first finding, related to the initial aim → a general relation between the average residual temperature <T> and the average temperature of initial fragments <T_i> *(A.Tudora et al. Eur.Phys.J A, 54 (2018) 87)*



$<T>_L \approx 0.6 <Ti>_L$ $<T>_H \approx 0.6 <Ti>_H$ $<T> \approx 0.6 <Ti>$

Irrespective of the prescriptions used for $\sigma_c(\varepsilon)$ and the level density parameters of initial and residual fragments



Replacement of the triangular P(T) with a moderately broad cut-off at high T by a distribution with a sharp cut-off.

This is justified by the use of a Weisskopf evaporation spectrum which overestimates somewhat the spectra at high energies. This overestimation can be compensated by a triangular P(T) with a sharp cut-off, which eliminates the residual temp. higher than Tmax = (3/2)<T>

$$P(T) = \begin{cases} 2T/T_{max}^2 & T \le T_{max} \\ 0 & T > T_{max} \end{cases}$$

$$<T> = \frac{2}{3} T_{max}$$

$$T_{max} = \frac{3}{2} 0.6 <Ti> = 0.9 <Ti>$$

Tudora et al., Eur.Phys.J.A 54 (2018) 87

$$T_{max} = <Ti>$$

$$<T> = \frac{2}{3} <Ti>$$

Madland and Nix

By solving $\overline{E_r}^{(k-1)} - S_n^{(k-1)} - <\varepsilon>_k = a_k T_k^2$ **for each A, Z, TKE**

different quantities for each emission sequence "k" → q_k(A,Z,TKE) are obtained, e.g. T_k(A,Z,TKE), Er_k(A,Z,TKE), $<\varepsilon>_k$(A,Z,TKE), the average energy carried away per each neutron η_k(A,Z,TKE) = $<\varepsilon>_k$(A,Z,TKE) + Sn_{k-1}(A,Z,TKE) etc.

They appear with the probability expressed by the Y(A,Z,TKE) distribution.

Average values corresponding to each emission sequence:

$$\langle q_k \rangle = \sum_{A,Z,TKE} q_k(A,Z,TKE)Y(A,Z,TKE) \Big/ \sum_{A,Z,TKE} Y(A,Z,TKE)$$

by summing separately for the light and heavy groups or over all fragments

A **total average quantity** corresponding to the sum of the distributions following the emission of each neutron is obtained by averaging $<q_k>$ over the probability for emission of each neutron (or the probability for apparition of each residual fragment) Pn_k:

$$\langle q \rangle = \sum_{k=1}^{n} \langle q_k \rangle Pn_k \Big/ \sum_{k=1}^{n} Pn_k$$

Note:

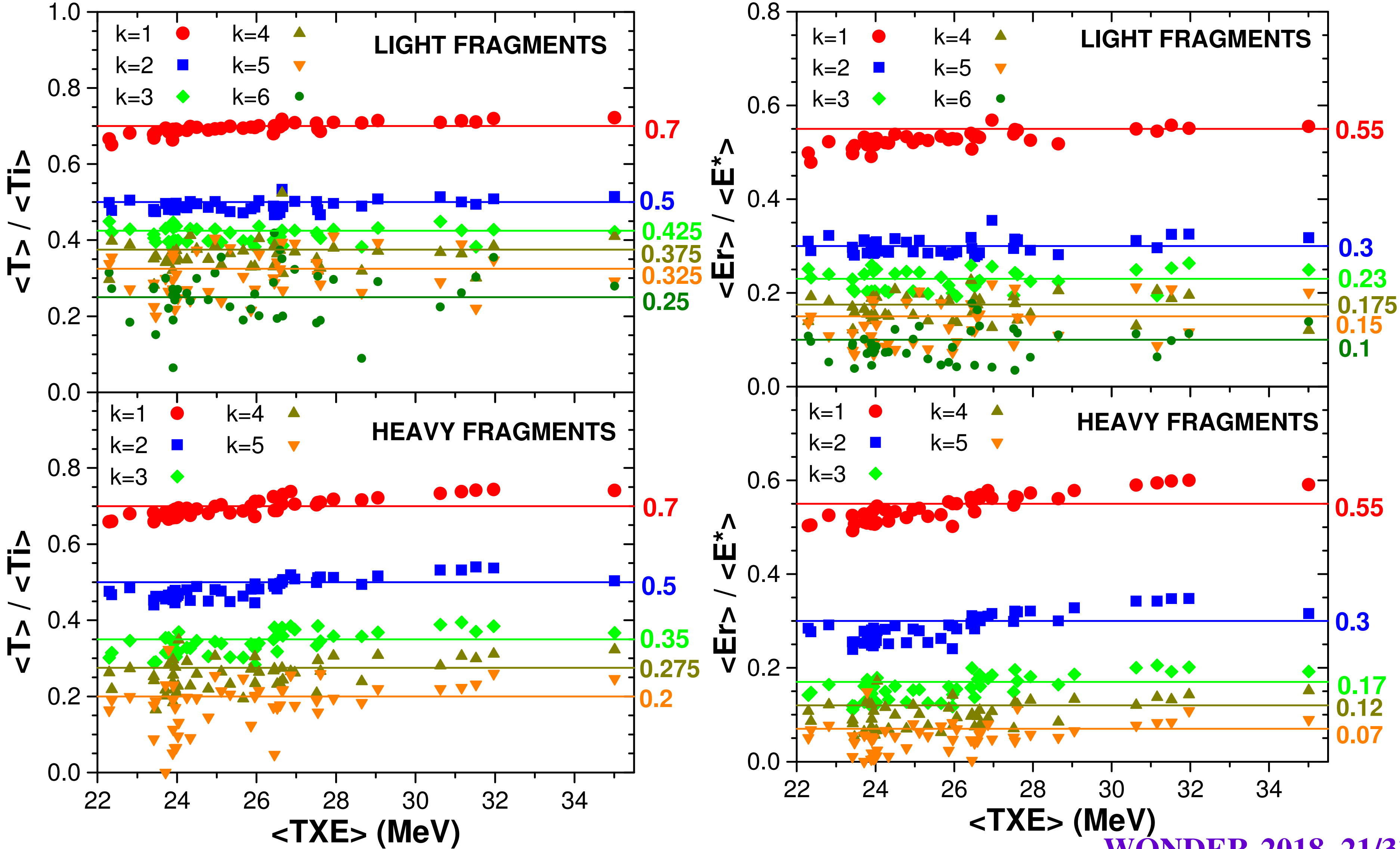
Pn_k = the probability for emission of the 1-st, second, …k-th neutron

to be not confounded with

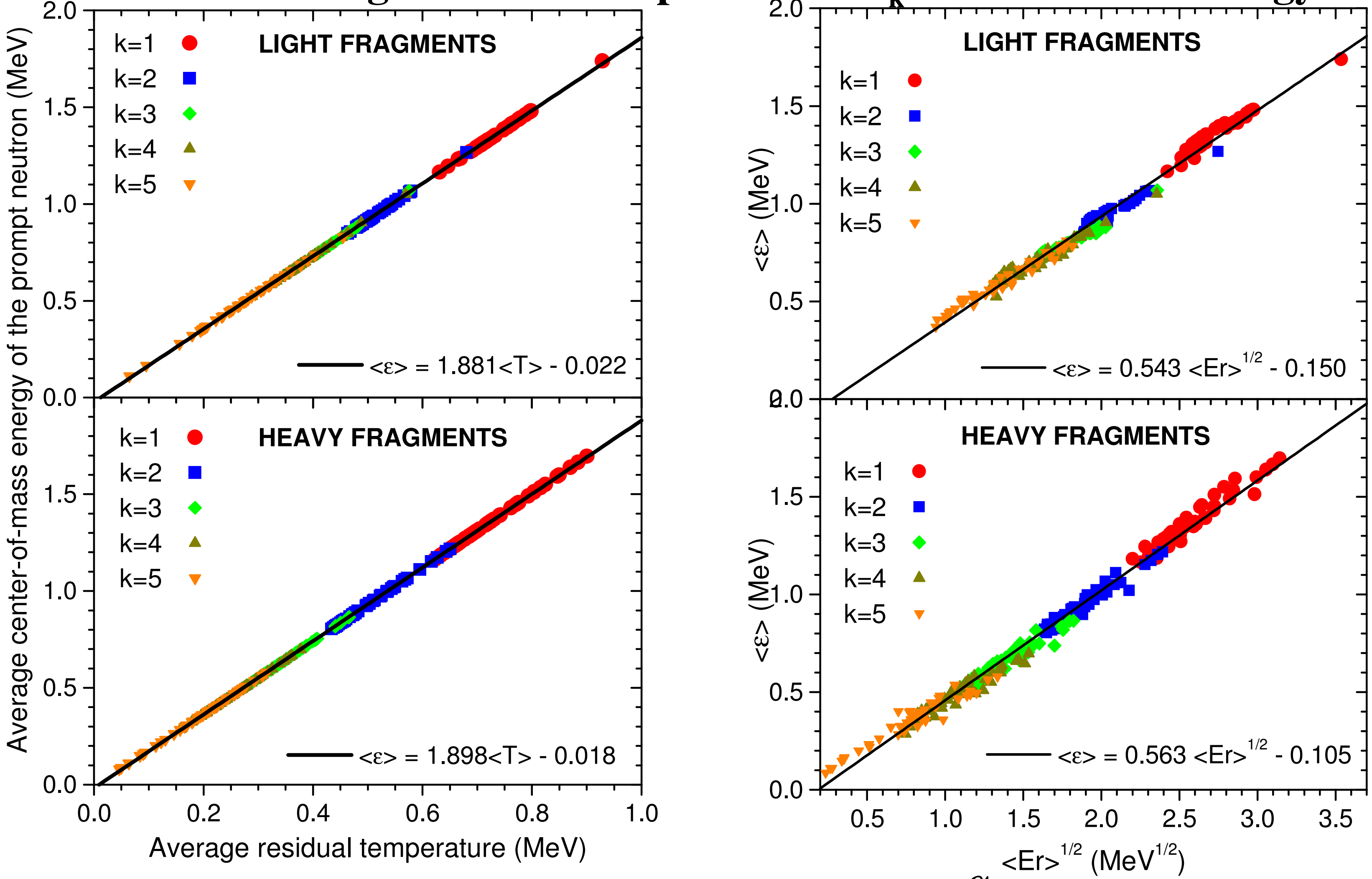
P(ν) = the probability for emission of one, two, three… neutrons

Ratios of residual temperatures and energies to the ones of initial fragments for the 49 studied fission cases

Prescriptions: analytical expression of $\sigma_c(\varepsilon)$ and level density parameters provided by the Egidy-Bucurescu systematic (2009) for BSFG



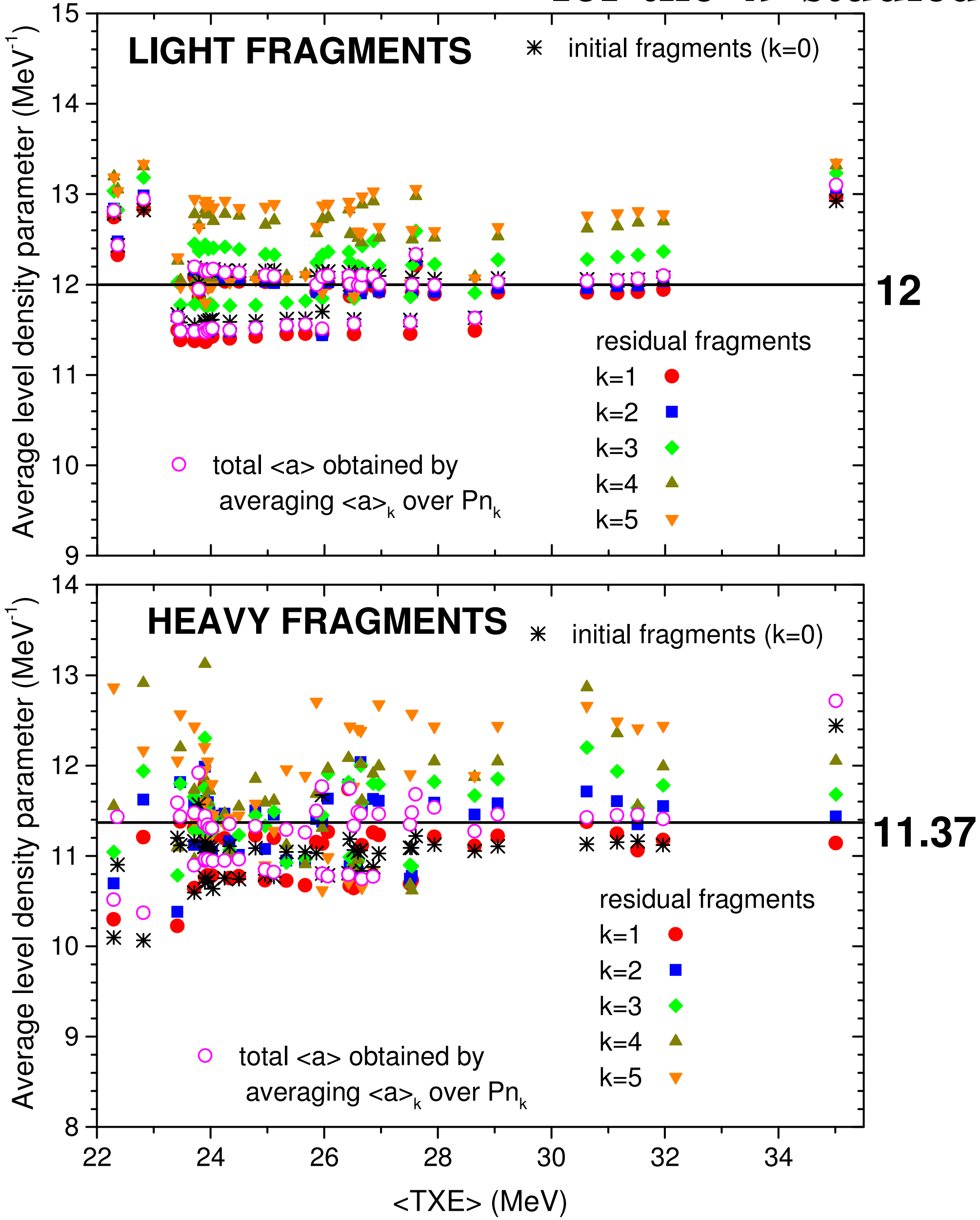
Average center-of-mass energy of prompt neutrons $<\varepsilon>_k$ as a function of average residual temperature $<T_k>$ and residual energy $<Er_k>^{1/2}$



from $<\varepsilon>=\alpha<T>+\beta$ and $<E_r>=<a><T>^2$ → $<\varepsilon>=\frac{\alpha}{\sqrt{<a>}}\sqrt{<E_r>}+\beta$

Using the slopes form these figures → global values of the average level density param. of the light and heavy fragm. groups: **$<a>_L$ = 12 MeV^{-1}, $<a>_H$=11.37 MeV^{-1}**

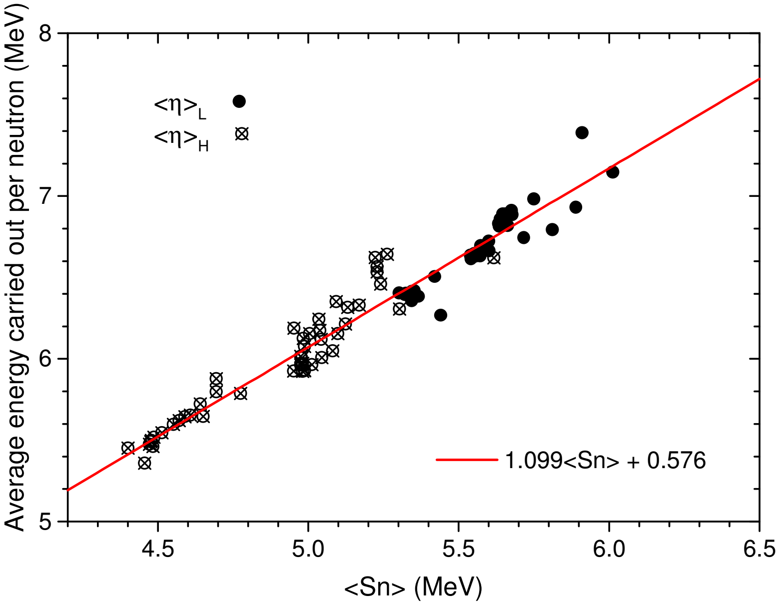
Average level density parameters of the initial and residual fragments for the 49 studied fission cases.

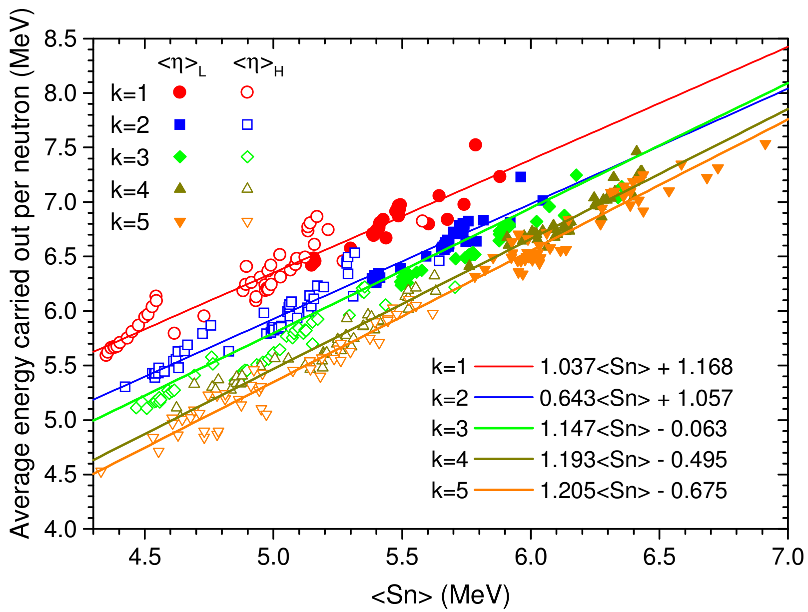


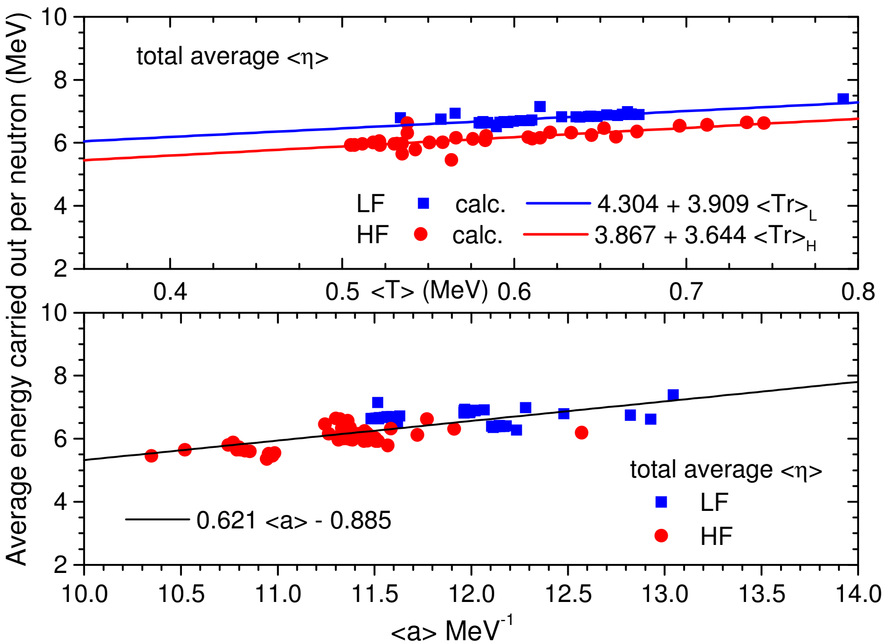
The global values of <a> (horizontal lines) resulting from the systematic behaviour of $<\varepsilon>_k$ as a func. of $<T>_k$ and $<Er_k>^{1/2}$ are in agreement with the total average <a> (magenta open circles).

The fact that <a> for **k=1** (red) and **k=2** (blue) **are close** to the **total** <a> (magenta open circles) is not surprising because the first two emission sequences take place for almost all fragments at the majority of TKE values.

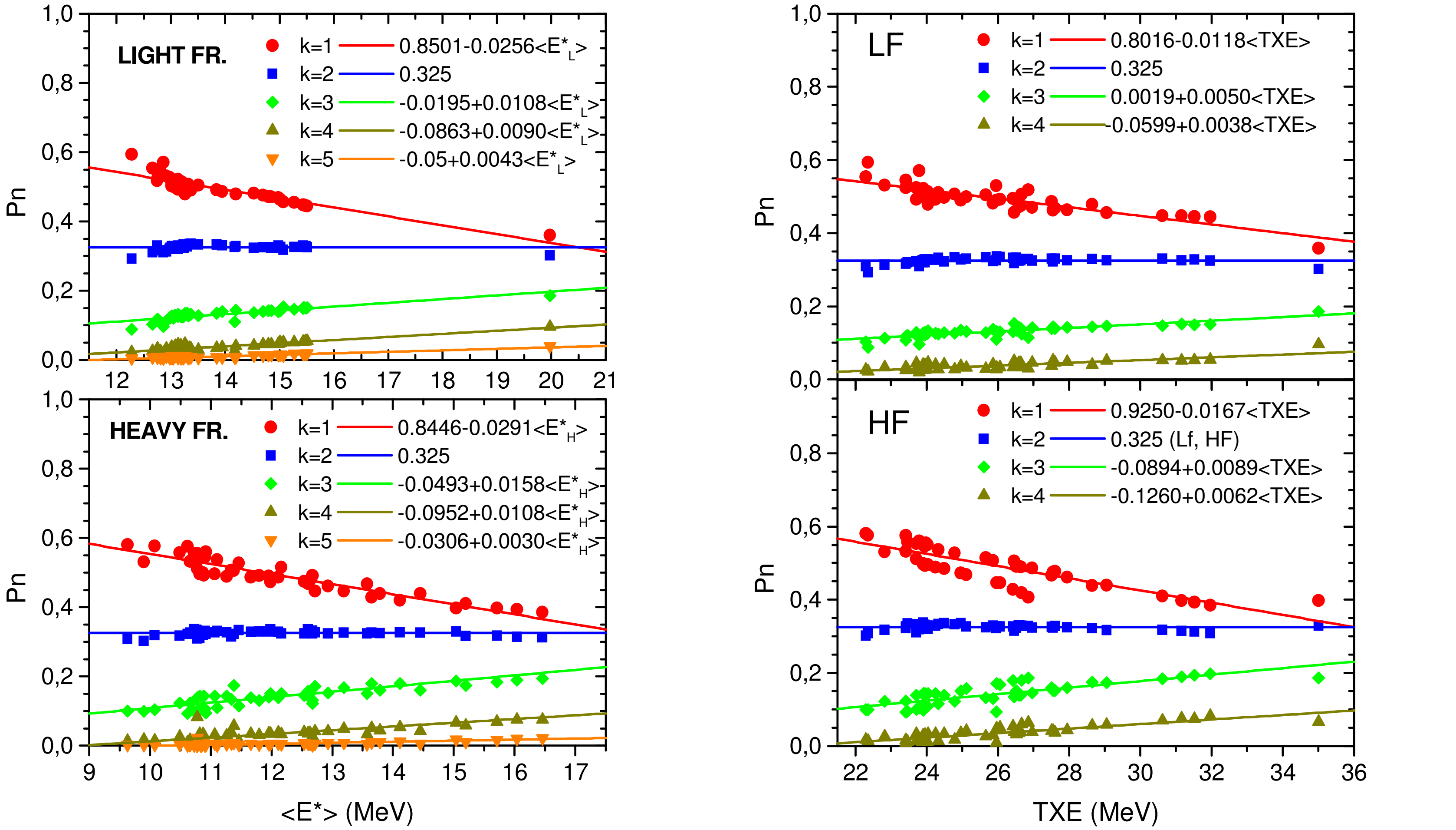
Energy carried away per neutron $\eta_k = <\varepsilon>_k + S_{n\ \text{precursor}}^{(k-1)}$





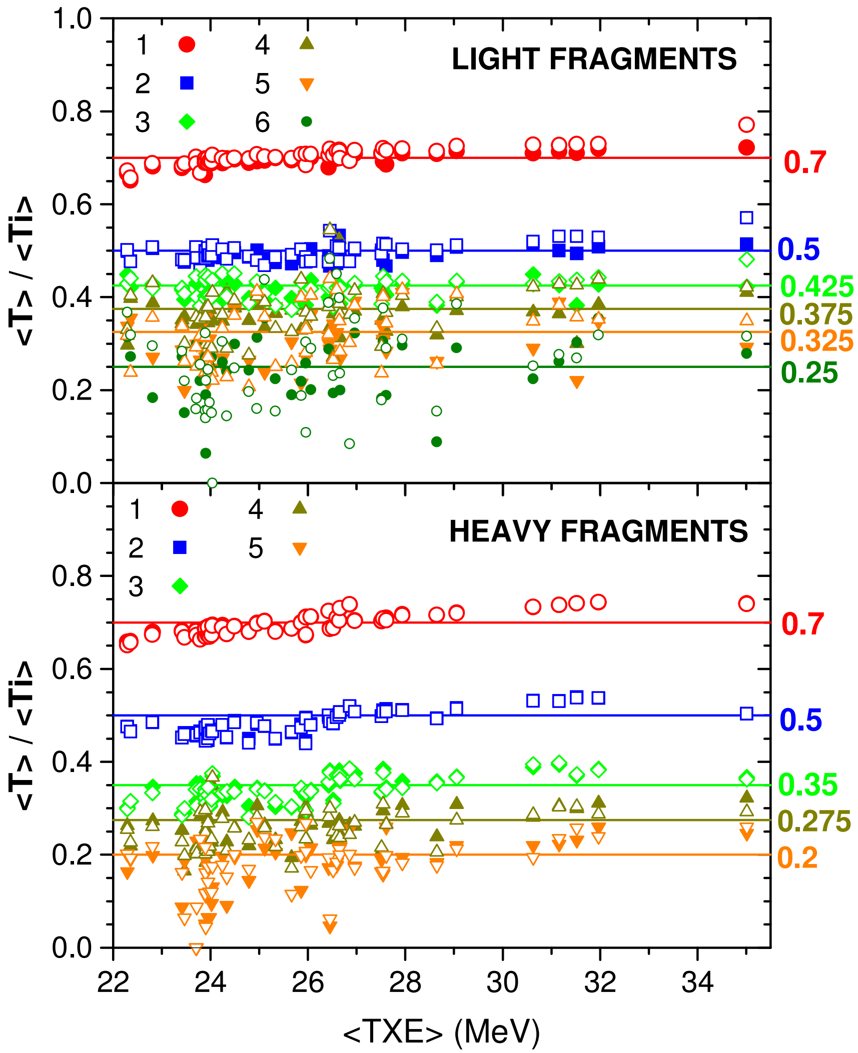


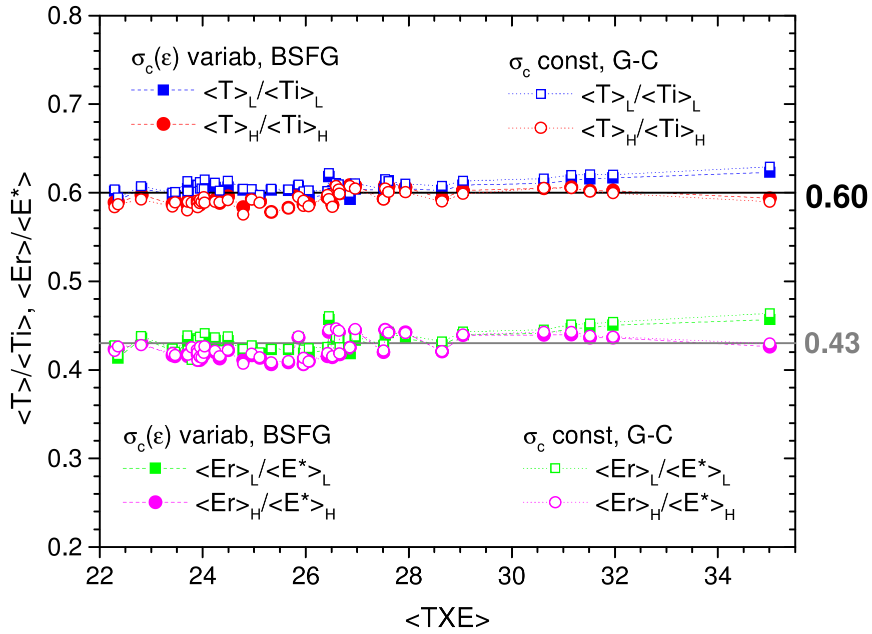
Probability for emission of the k-th prompt neutron from the light and heavy fragment groups as a function of the average excitation energy of the initial light and heavy fragments and as a function of <TXE> for the 49 studied fission cases.



The use of other prescriptions does not change the results.

Here examples for the prescriptions $\sigma_c(\varepsilon)$=constant and level dens. parameters of the Gilbert-Cameron systematic for spherical nuclei, which are very different from the prescriptions previously employed (i.e. analytical expression of $\sigma_c(\varepsilon)$, lev.dens.param. provided by the systm. E-B 2009 for BSFG)





Full symbols: analytical expression $\sigma_c(\varepsilon)$ and E-B 2009 systm. for BSFG

Open symbols: constant $\sigma_c(\varepsilon)$ and G-C systematic

APPLICATION of the systematic behaviour $<T>_k/<T_i>=r_k$

Inclusion of the sequential emission into the Los Alamos model

Up to now in the LA model → Tmax of P(T) was taken equal to <Ti>

$$<Ti> = \sqrt{<TXE>/<a_L + a_H>}$$

Madland and Nix (NSE 1982) **the same P(T)**

$$<Ti>_{L,H} = \sqrt{<E^*>_{L,H} / <a>_{L,H}}$$

Madlald and Kahler (NPA 2017)

non-equal Tmax for LF and HF (as in PbP)

Now:

The consideration of a triangular $P_k(T)$ for each emission sequence "k"

with:

$$T_{\max}^{(k)} = \frac{3}{2} r_k < Ti >$$

$$P_k(T) = \begin{cases} 2T/T_{\max}^{(k)2} & T \le T_{\max}^{(k)} \\ 0 & T > T_{\max}^{(k)} \end{cases}$$

r_k given by the systematic,

e.g. r_1=0.7, r_2=0.5 etc.

c.m.s.

$$\Phi_k(\varepsilon) = \int_0^{T\max^{(k)}} \varphi(\varepsilon,T)\, P_k(T)\, dT = \varepsilon\, \sigma_c^{(k)}(\varepsilon) \int_0^{T\max^{(k)}} K_k(T)\, P_k(T) \exp(-\varepsilon/T)\, dT$$

$$K_k(T) = \left(\int_0^{\infty} \varepsilon\, \sigma_c^{(k)}(\varepsilon) \exp(-\varepsilon/T)\, d\varepsilon \right)^{-1}$$

Inclusion of the sequential emission into the Los Alamos model

For the input parameters of the LA model (as average values), different prescriptions can be used regarding:

a) $\sigma_c(\varepsilon)$: constant or an analytical expression (depending on the mass number and the s-wave neutron strength function of the nucleus {Z, A-k+1} or provided by optical model calc. with phenomenological potentials adequate for nuclei appearing as FF

b) TXE partition: e.g. by modeling at scission (PbP), the procedure proposed by Madland and Kahler, the method of FREYA (adjustable param."x") of FIFRELIN (implying the nucl.temp.ratio RT)) etc.

c) level density parameters of fragments: either energy-dependent (super-fluid with different shell corrections and parameterizations of the dumping and asymptotic lev. dens.) or non-energy dependent (e.g. systematics of EB-2009 for BSFG, G-C etc.)

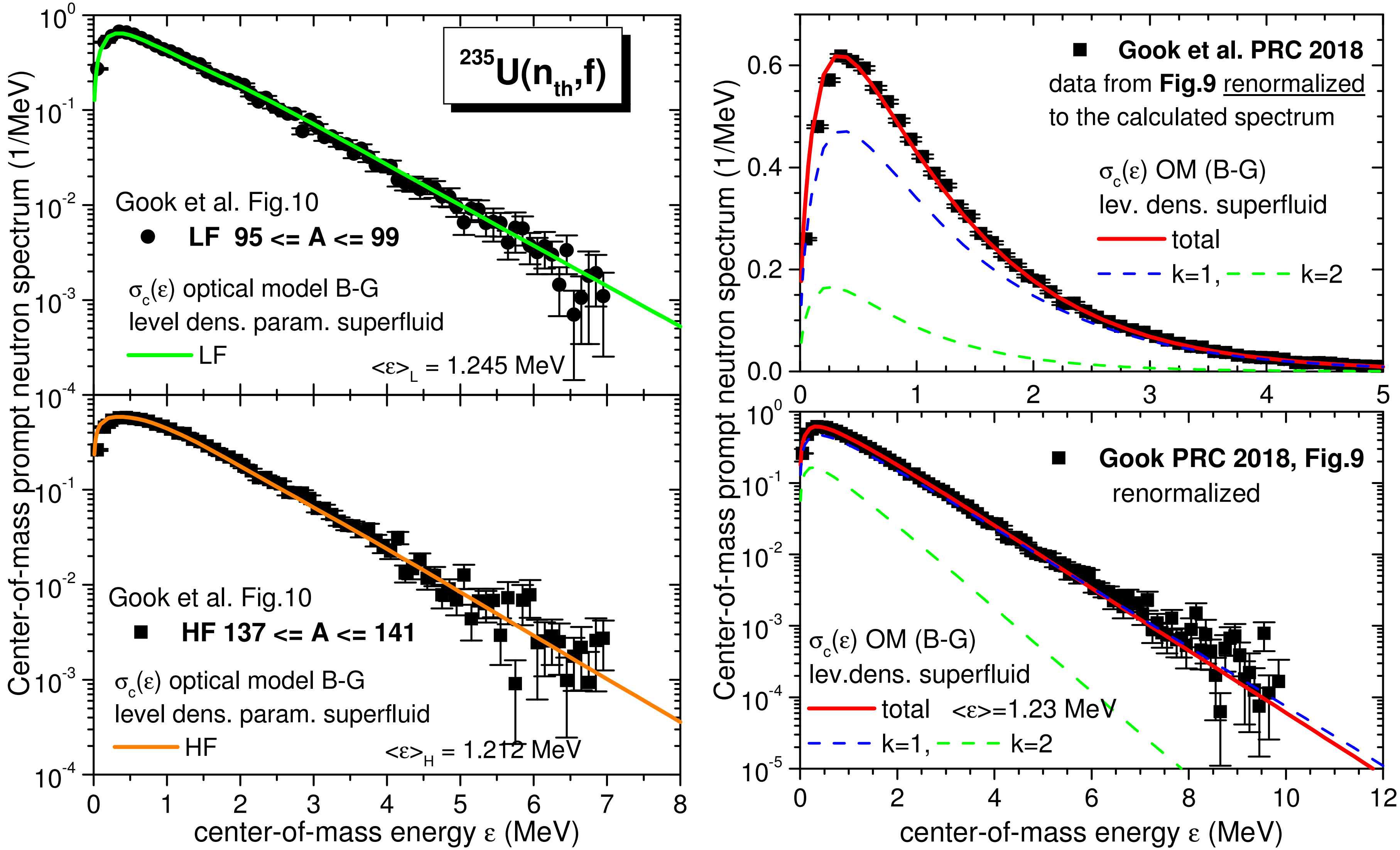
Example: $\langle \varepsilon \rangle_k = \int_0^\infty \varepsilon \Phi_k(\varepsilon) d\varepsilon$ $\quad \langle \varepsilon \rangle = \sum_{k=1}^{\bar{n}} Pn_k \langle \varepsilon \rangle_k \Big/ \sum_{k=1}^{\bar{n}} Pn_k$

^{252}Cf(SF)	$<\varepsilon>_L$	$<\varepsilon>_H$	$<\varepsilon>$	exp (Göök)	
i) $\sigma_c(\varepsilon)$ const, EB-2009 BSFG	1.432	1.254	1.356	~1.45	(6%)
ii) $\sigma_c(\varepsilon)$ OM (B-G), super-fluid	1.517	1.309	1.428		(1.5%)
average number of sequences n=3					

Example: $$\Phi_{L,H}(\varepsilon) = \sum_{k=1}^{\bar{n}} Pn_k \, \Phi_k^{(L,H)}(\varepsilon) \Big/ \sum_{k=1}^{\bar{n}} Pn_k$$

sequential emission into the Los Alamos model

Prescriptions: $\sigma_c(\varepsilon)$ optical model B-G, TXE partition by modeling at scission and level density parameters of the super-fluid model



The systematic behaviours presented above can be also used in order to obtain indicative values of different average prompt emission quantities <u>in the absence of any prompt emission model</u>.

If the average temperatures of initial fragments are known for a given fissioning nucleus (i.e. the equivalent $\langle Ti \rangle$ or $\langle Ti \rangle_L$ and $\langle Ti \rangle_H$)

then $\langle \varepsilon \rangle$ can be obtained from the linear behaviour presented above i.e. $\langle \varepsilon \rangle_L = 1.881\langle T \rangle - 0.022$ and $\langle \varepsilon \rangle_H = 1.898\langle T \rangle - 0.018$ by using the ratio $\langle T \rangle / \langle Ti \rangle \sim 0.6$.

Examples for ^{252}Cf(SF) for which

$\langle TXE \rangle = 35.01$ MeV, $\langle E^* \rangle_L = 19.973$ MeV and $\langle E^* \rangle_H = 15.037$ MeV

i) using the equivalent $\langle Ti \rangle$ based on $\langle a \rangle = A_0/11$ MeV^{-1} → $\langle \varepsilon \rangle = 1.382$ MeV

ii) considering $\langle Ti \rangle_{L,H}$ based on level dens.param. of the supefluid model $\langle a \rangle_L = 13.545$ MeV^{-1}, $\langle a \rangle_H = 12.759$ MeV^{-1}

→ $\langle \varepsilon \rangle_L = 1.430$ MeV, $\langle \varepsilon \rangle_H = 1.273$ MeV, $\langle \varepsilon \rangle = 1.363$ MeV

which deviate with 0.7% from the result of Madland and Kahler (NPA 2017)

CONCLUSIONS

The deterministic treatment of sequential emission applied to <u>49 fission cases</u> allowed to obtain systematic behaviours and correlations between different average quantities characterizing the initial and residual fragments and the prompt neutron emission

1. The ratios $\langle T\rangle/\langle Ti\rangle$ of LF, HF groups and of all fragments <u>are of about 0.6</u> irrespective of the prescriptions used for $\sigma_c(\varepsilon)$ and the lev.dens.parameters, leading to a triangular P(T) with $T_{max} = 0.9 \langle Ti\rangle$ (*A.Tudora et al. EPJA 54 (2018)*)

- **$\langle T\rangle_k/\langle Ti\rangle = r_k$ (e.g. r_1=0.7, r_2=0.5 for LF, HF, r_3=0.425 (LF), 0.35 (HF) etc.) allow to define $P_k(T)$ for each emission sequence with $T_{max}^{(k)} = (3/2) r_k \langle Ti\rangle$ and the <u>inclusion of sequential emission into the Los Alamos model</u>.**

2. The constant ratios $\langle T\rangle/\langle Ti\rangle$~0.6 and the linear behaviour of $\langle \varepsilon\rangle_{L,H}$ as a function of $\langle T\rangle_{L,H}$ allow to obtain indicative values of different average prompt emission quantities in the absence of any prompt emission model.

3. $\langle Er\rangle/\langle E^*\rangle$ of LF, HF groups and of all FF are of about 0.43 irrespective of the prescriptions mentioned above and also $\langle Er\rangle_k/\langle E^*\rangle = r'_k$ (r'_1=0.55, r'_2=0.3 etc.)

4. The linear dependences of $\langle \varepsilon\rangle_k$ on $\langle T\rangle_k$ and on $(\langle Er\rangle_k)^{1/2}$ are the same for all emission sequences. Almost linear dependences of $\langle \eta\rangle_k$ on $\langle Sn\rangle_{k-1}$, $\langle T\rangle_k$ and $\langle a\rangle_k$ are established, too.

Many thanks for your attention