

# Local and global even-odd effects in prompt emission in fission

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**Abstract.** The investigation of the proton even-odd effects in prompt emission in fission for even- $Z$  actinides revealed basic features of the global even-odd effect in prompt emission similar with those in fission fragment yields and some particular aspects, such as: (1) the even-odd effects in prompt emission are the result of two contributions: a dominant intrinsic even-odd effect due to the even-odd nuclear character of fragments reflected in their properties and a weak even-odd effect caused by the fragment distributions (over which the multi-parametric matrices are averaged); (2) oscillations with a periodicity of about 5 mass units are present in different prompt emission quantities corresponding to even- $Z$  and odd- $Z$  fragmentations independent on the size of the even-odd effect in the charge yield  $Y(Z)$ . These oscillations are due to the periodicity of nuclear properties of fragments; (3) a local even-odd effect in prompt emission quantities has been recently investigated. Similarities between prompt emission quantities and fragment yields were found in the case of the local even-odd effect, too. The local even-odd effect in both fragment charge yields and prompt emission quantities exhibit a pronounced increase at asymmetry values corresponding to fragmentations in which the heavy fragment ( $Z = 50$  and/or  $N = 82$ ) or the light one ( $Z = 28$ ) is magic.

## 1. Introduction

Lately [1–5] our attention was turned to a subject, not yet investigated, namely proton ( $Z$ ) and neutron ( $N$ ) even-odd effects in prompt emission in fission. We have studied even- $Z$  nuclei fissioning spontaneously or induced by thermal neutrons, like  $^{233,235}\text{U}(n,f)$ ,  $^{234}\text{U}(n,f)$ ,  $^{239}\text{Pu}(n_{th},f)$ ,  $^{236,238,240,242,244}\text{Pu}(SF)$ ,  $^{252}\text{Cf}(SF)$ , because the extensive studies (e.g., [6–9]) regarding the  $Z$  even-odd effect in fragment distributions showed that the effect is most pronounced for this type of nuclei.

This topic is of major importance for a more profound understanding of the nuclear fission process, for the determination of the fragment distributions (which depends on knowing with high accuracy the prompt emission data, including even-odd effects).

The prompt emission calculations were done in the frame of the Point-by-Point (PbP) model (described in [10] and references therein). The primary results of the PbP model are the multi-parametric matrices of different quantities characterizing the fission fragments and the prompt emission, generally labelled as  $q(A,Z,TKE)$  (e.g., prompt neutron multiplicity  $\nu(A,Z,TKE)$ , prompt  $\gamma$ -ray energy  $E_\gamma(A,Z,TKE)$ ). Average quantities as a function of  $Z$  ( $q(Z)$ ), of  $A$  ( $q(A)$ ), of  $TKE$  ( $q(TKE)$ ) and total average  $\langle q \rangle$  are obtained by averaging the PbP matrices over the fragment distributions  $Y(A,Z,TKE)$  in different ways (details are given in Refs. [1, 2, 10] and references therein). These distributions are constructed as  $Y(A,Z,TKE) = Y(A,TKE) p(Z,A)$  in which  $Y(A,TKE)$  are experimental distributions (usually reconstructed from the single ones  $Y(A)$ ,  $TKE(A)$  and  $\sigma_{TKE}(A)$ ) and the isobaric charge

distributions  $p(Z,A)$  are provided by the  $Z_p$  model of Wahl [11–13].

We started to study the basic features of the even-odd effect in prompt emission quantities, e.g., the behaviour of different average quantities like  $q(Z)$ ,  $q(A)$  of even- $Z$  and odd- $Z$  fragmentations, the behaviour of the global  $Z$  even-odd effect in different total average quantities, defined as [1, 2]:

$$\delta_{\langle q \rangle} = \frac{\langle q \rangle_{\text{even-Z}} - \langle q \rangle_{\text{odd-Z}}}{\langle q \rangle} \quad (1)$$

where  $\langle q \rangle_{\text{even-Z}}$ ,  $\langle q \rangle_{\text{odd-Z}}$  and  $\langle q \rangle$  are any quantities corresponding to even- $Z$ , to odd- $Z$  and to all- $Z$  fragmentations, respectively.

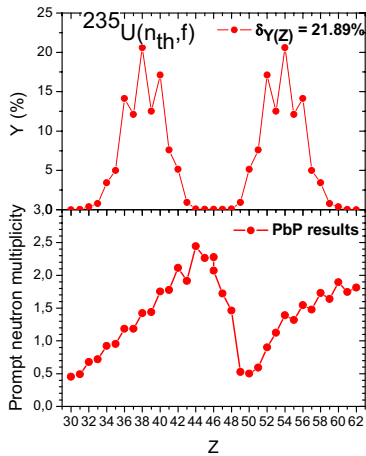
Recently, we have investigated some particular aspects related to the even-odd effects in prompt emission [5], like: the periodicity of five mass units in average quantities as a function of fragment mass, the intrinsic even-odd effect of prompt emission, the local  $Z$  even-odd effect in prompt neutron multiplicity and TXE.

## 2. Basic features of the global $Z$ even-odd effect in prompt emission

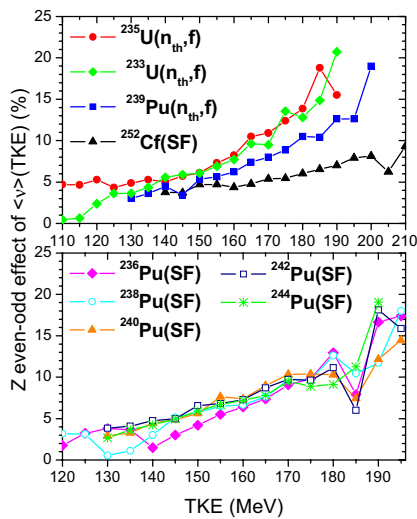
The main features of the global  $Z$  even-odd effect in prompt emission [1–3] are similar with those in fission fragment charge yields  $Y(Z)$  [6–9]:

- (1) the global  $Z$  even-odd effect in prompt emission decreases with increasing mass of the fissioning nucleus, e.g., from 9% ( $^{233,235}\text{U}(n_{th},f)$ ) to about 6% ( $^{252}\text{Cf}(SF)$ ). The global  $Z$  even-odd effect in  $Y(Z)$  decreases from about 21% ( $^{236}\text{U}$ ) to 4% ( $^{252}\text{Cf}(SF)$ ).

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**Figure 1.**  $^{235}\text{U}(n_{\text{th}}, f)$ :  $Y(Z)$  projection (upper part) and prompt neutron multiplicity as a function of  $Z$  (lower part).



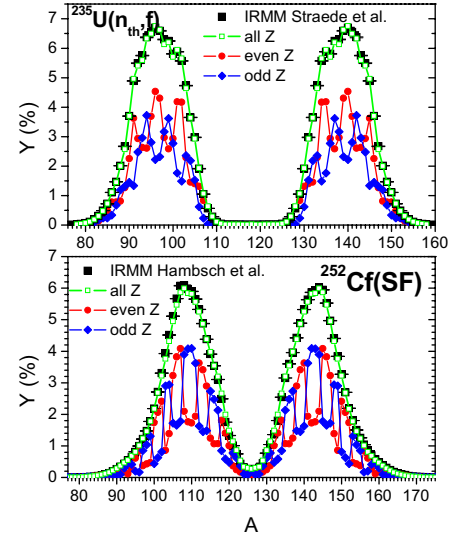
**Figure 2.**  $Z$  even-odd effect in  $\langle \nu \rangle$  (TKE) for:  $^{235,233}\text{U}(n_{\text{th}}, f)$ ,  $^{239}\text{Pu}(n_{\text{th}}, f)$ ,  $^{252}\text{Cf}(\text{SF})$  (upper part) [1] and  $^{236}\text{Pu}(\text{SF})$ ,  $^{238}\text{Pu}(\text{SF})$ ,  $^{240}\text{Pu}(\text{SF})$ ,  $^{242}\text{Pu}(\text{SF})$  and  $^{244}\text{Pu}(\text{SF})$  (lower part) [2].

- (2) prompt neutron multiplicity as a function of  $Z$ ,  $\nu(Z)$ , exhibit a visible staggering in the asymmetric fission region as  $Y(Z)$  does. This fact can be seen in the examples given in Fig. 1.
- (3) the  $Z$  even-odd effect increases with increasing kinetic energy of the fission fragments. In the case of prompt emission, this fact is emphasized by the following function:  $\delta_{\langle q \rangle}(\text{TKE}) = (\langle q \rangle_e(\text{TKE}) - \langle q \rangle_o(\text{TKE})) / \langle q \rangle(\text{TKE})$  introduced in Ref. [1]. An example of this function is given in Fig. 2 for  $\langle \nu \rangle$  (TKE) of  $^{233,235}\text{U}(n_{\text{th}}, f)$ ,  $^{239}\text{Pu}(n_{\text{th}}, f)$  and  $^{252}\text{Cf}(\text{SF})$  (upper part) and  $^{236,238,240,242,244}\text{Pu}(\text{SF})$  (lower part).

### 3. Particular aspects related to even-odd effects in prompt emission

We have seen in our studies that average quantities as a function of  $A$  corresponding to even- $Z$  and to odd- $Z$  fragmentations exhibit oscillations with a periodicity of about 5 mass units [1–5].

The same periodicity was seen in experimental data of charge polarization  $\Delta Z(A)$  and the root-mean-square



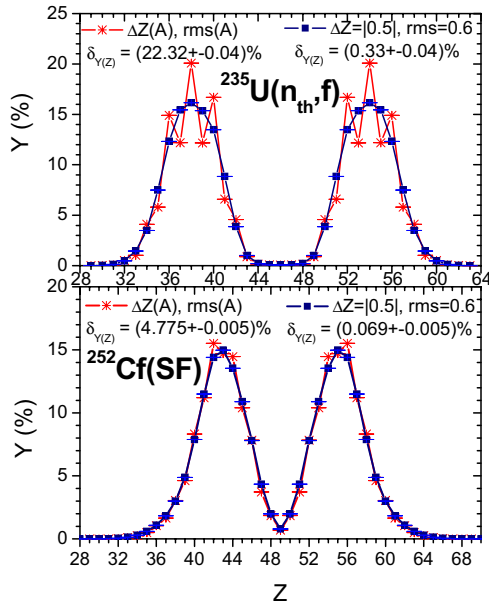
**Figure 3.**  $Y(A)$  of even- $Z$  fragmentations (red circles) and odd- $Z$  fragmentations (blue diamonds),  $Y(A)$  of all fragmentations (open green squares) and experimental  $Y(A)$  data (full black squares) for the fissioning systems  $^{235}\text{U}(n_{\text{th}}, f)$  (upper part) and  $^{252}\text{Cf}(\text{SF})$  (lower part).

$\text{rms}(A)$  of the isobaric charge distribution  $p(Z, A)$ , well described by the  $Z_p$  model of Wahl [11–13], in the asymmetric fission region [6]. Gönnerwein has made a connection between the oscillations of  $\Delta Z(A)$  and  $\text{rms}(A)$  with a period  $\Delta A \approx 5$  and the presence of even-odd effects in charge yield  $Y(Z)$  [6].

In the case of  $\Delta Z(A)$ ,  $\text{rms}(A)$  and  $Y(A)$  of even- $Z$  and odd- $Z$  fragments, only the magnitude of the oscillations amplitudes is related to the size of the even-odd effect in  $Y(Z)$ . This fact can be seen in the example given in Fig. 3 where  $Y(A)$  is plotted separately for even- $Z$  (red circles), odd- $Z$  (blue diamonds) and all- $Z$  fragmentations (black squares) for two fissioning nuclei (the extreme fissioning systems in terms of the size of the even-odd effect in  $Y(Z)$ ):  $^{235}\text{U}(n_{\text{th}}, f)$  (upper part) and  $^{252}\text{Cf}(\text{SF})$  (lower part). It can be observed that  $Y(A)$  of even- $Z$  and odd- $Z$  fragmentations oscillate in anti-phase with a period  $\Delta A \approx 5$ . It can be also seen that in the case of  $^{235}\text{U}(n_{\text{th}}, f)$  (for which the global even-odd effect in  $Y(Z)$  is high, of about 22%) the amplitudes of the oscillations are visibly higher for even- $Z$  fragmentations than for odd- $Z$  ones while for  $^{252}\text{Cf}(\text{SF})$  (with a lower  $\delta_{Y(Z)}$ , of about 4%) the amplitudes of the oscillations are almost equal. Thus, higher amplitudes of  $Y(A)$  of even- $Z$  fragmentations compared to those of odd- $Z$  fragmentations means the presence of the even-odd effect in  $Y(Z)$ . At limit, equal amplitudes in anti-phase cancel the even-odd effect in  $Y(Z)$ .

In the case of  $\Delta Z(A)$  and  $\text{rms}(A)$  the magnitude of their oscillation amplitudes is proportional with the size of the even-odd effect in  $Y(Z)$ . At limit, zero amplitude, i.e., no oscillation of  $\Delta Z(A)$  and  $\text{rms}(A)$ , means no even-odd effect in  $Y(Z)$  (details are given in Ref. [5])

Regarding the prompt emission quantities, the oscillations with the period  $\Delta A \approx 5$  persists even when fragment distributions without even-odd effects are used to obtain different average quantities. This fact was shown in Ref. [5], where relevant quantities for prompt emission, such as the energy release ( $Q$ -value) and the total excitation energy of fully accelerated fragments



**Figure 4.**  $Y(Z)$  projections obtained in two cases of using  $\Delta Z$  and rms:  $\Delta Z = |0.5|$  and rms = 0.6 (navy squares),  $\Delta Z(A)$  and rms(A) (red stars) [5].

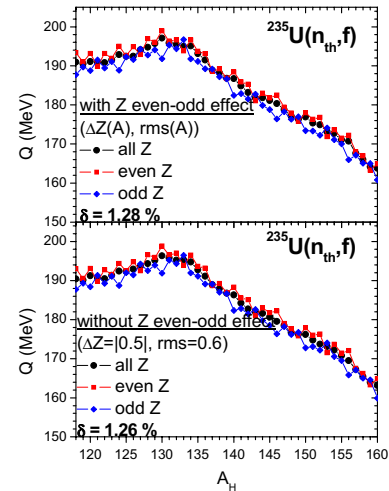
(TXE), were averaged over two types of  $Y(A,Z,TKE)$  distributions: one with even-odd effects (constructed by taking in the Gaussian expression of  $p(Z,A)$  oscillating  $\Delta Z(A)$  and rms(A)) and one without even-odd effects (constructed by considering for all fragments  $\Delta Z = |0.5|$  and the same value of 0.6 for the rms of  $p(Z,A)$ ).

In Fig. 4 are given examples of  $Y(Z)$  projections for  $^{235}\text{U}(n_{th},f)$  (upper part) and  $^{252}\text{Cf}(SF)$  (lower part), obtained in the two cases of  $Y(A,Z,TKE)$  distributions. As it can be seen, the even-odd effect in  $Y(Z)$  disappears when constant  $\Delta Z$  and rms are used, being reflected in the lack of  $Y(Z)$  staggering and in almost equal to zero global  $Z$  even-odd effect (given in the legend) [5].

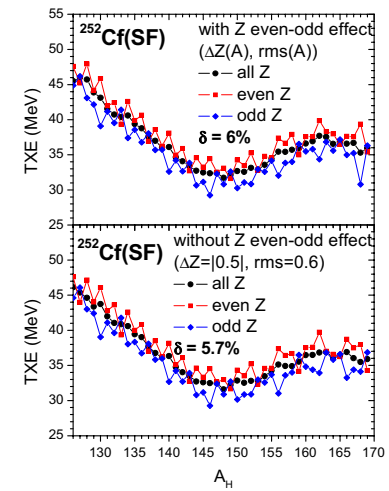
Examples of  $Q(A)$  and  $TXE(A)$ , obtained by averaging over the two types of distributions, are plotted in Fig. 5 ( $^{235}\text{U}(n_{th},f)$ ) and 6 ( $^{252}\text{Cf}(SF)$ ), respectively.

For both fissioning systems  $Q(A)$  and  $TXE(A)$  of even- $Z$  and odd- $Z$  fragmentations exhibit oscillations with a periodicity of about 5 mass units in both cases of  $Y(A,Z,TKE)$  (with or without even-odd effect). This fact proves that the periodicity of these oscillations is independent of the existence or not of an even-odd effect in charge distributions, being a consequence of the periodicity in the nuclear properties of the fragments (e.g., mass excesses, binding energies, pairing energies).

The values of the global  $Z$  even-odd effect in  $\langle TXE \rangle$ , (and  $\langle Q \rangle$ ), given in the legends of Figs. 5 and 6, are almost the same in both cases of fragment distributions (with and without  $Z$  even-odd effect). This fact together with the important role played by  $TXE$  in the prompt emission has demonstrated that the even-odd effects in different quantities related to the prompt emission are mainly due to the nuclear properties of fission fragments. Consequently, the even-odd effects in prompt emission are the result of two contributions [4,5]: (i) a dominant intrinsic even-odd effect due to the even-odd nuclear character of fragments reflected in their properties (and consequently in the emitted prompt neutrons and  $\gamma$ -rays)



**Figure 5.**  $^{235}\text{U}(n_{th},f)$ : average  $Q(A)$  obtained by averaging  $Q(A,Z)$  over  $Y(A,Z,TKE)$  with  $Z$  even-odd effect (upper part) and another one without  $Z$  even-odd effects (lower part) [5].



**Figure 6.**  $^{252}\text{Cf}(SF)$ : average  $TXE(A)$  obtained by averaging  $TXE(A,Z,TKE)$  over  $Y(A,Z,TKE)$  with  $Z$  even-odd effect (upper part) and without  $Z$  even-odd effects (lower part) [5].

and (ii) a weak even-odd effect caused by the fragment distributions.

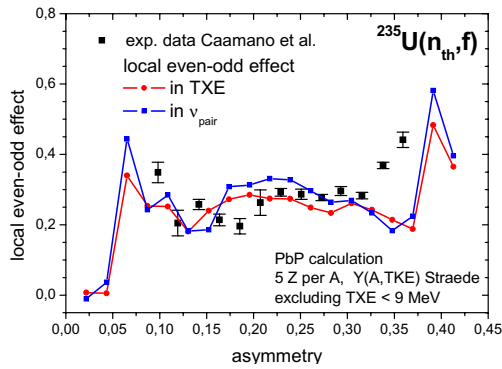
The dominance of the intrinsic even-odd effect was also demonstrated by the even-odd nucleus  $^{234}\text{U}(n,f)$  at 14 incident neutron energies ranging from 0.2 MeV to 5 MeV (see Ref. [4]).

#### 4. Local even-odd effect in prompt emission in fission

The behavior of different quantities corresponding to the four possible types of fragmentation of a fissioning nucleus (i.e., even-even, even-odd, odd-even and odd-odd for an even-even fissioning nucleus) [1–5] suggested the possibility of defining, for the first time, a local even-odd effect in prompt emission quantities (generally labelled “ $q$ ”), as [5]:

$$\delta_{p\langle q \rangle} = \frac{1}{2} \frac{\langle q \rangle_{\text{even-Z}} - \langle q \rangle_{\text{odd-Z}}}{\langle q \rangle_{\text{even-Z}} + \langle q \rangle_{\text{odd-Z}}} \quad (2)$$

where  $\langle q \rangle_{\text{even-Z}}$  and  $\langle q \rangle_{\text{odd-Z}}$  are normalized quantities corresponding to even- $Z$  and to odd- $Z$  fragmentations.



**Figure 7.** Local even-odd effect in TXE (red circles) and prompt neutron multiplicity of fragment pair (blue squares) for  $^{235}\text{U}(n_{\text{th}}, f)$  in comparison with the experimental data of the local even-odd effect in fragment distributions of Caamano et al. [9] (black squares) plotted as a function of asymmetry parameter [5].

In Fig. 7 are given examples of local even-odd effect in TXE (red circles) and prompt neutron multiplicity of fragment pair (blue squares) as a function of asymmetry parameter, defined as  $a_s = (Z_H - Z_L)/Z_0$ , for  $^{235}\text{U}(n_{\text{th}}, f)$ .  $\delta_{p-\langle v_{\text{pair}} \rangle}$  is a little bit higher than  $\delta_{p-\langle \text{TXE} \rangle}$  fact confirmed by the slightly higher global even-odd effect for  $\langle v \rangle$  compared to  $\langle \text{TXE} \rangle$  [1]. An interesting observation is that the local even-odd effect in both TXE and  $v_{\text{pair}}$  exhibits a similar behaviour as the experimental data of Caamaño et al. [14] (black squares) concerning the local even-odd effect in fragment distributions [5].

The pronounced increase of the local even-odd effect at asymmetry values corresponding to fragmentations in which one of the fragment is magic or double magic (seen in Fig. 7 at asymmetry values of about 0.7 corresponding to fragmentations with magic heavy fragment ( $N_H = 82$ ) and around 0.4 corresponding to very asymmetric fragmentations in which the light fragment is magic ( $Z_L = 28$ )) is a consequence of the important role played by the fragment properties reflecting the even-odd nuclear character of fragments (i.e., the contribution of the intrinsic even-odd effect).

## 5. Conclusions

The basic features of the even-odd effect in prompt emission are similar with those in fragment yields.

The periodicity  $\Delta A \approx 5$  of the oscillations in the charge polarization  $\Delta Z(A)$ , rms(A) of the isobaric charge distribution, as well as in the fragment mass yields  $Y(A)$  and different quantities related to the prompt emission corresponding to even-Z and odd-Z fragmentations are due

to the periodicity of nuclear properties of fragments, being independent of the presence or not of the even-odd effect in the charge yield  $Y(Z)$ .

The even-odd effect in prompt emission quantities is the result of two contributions: a dominant intrinsic even-odd effect due to the nuclear properties of fragments and a weaker even-odd effect brought by the fragment distributions (over which the multi-parametric matrices are averaged).

The local even-odd effect in TXE and prompt neutron multiplicity exhibits the same behavior as the local even-odd effect in fragment yields. The feature of the local even-odd effect, consisting in a pronounced increase at asymmetry values corresponding to fragmentations in which the heavy fragment ( $Z = 50$  and/or  $N = 82$ ) or the light one ( $Z = 28$ ) is magic, is present in both the charge yield and prompt emission quantities.

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